Assignment

Subject - Mathematics

Semester - 4, Core Course - 8

Topic - Improper Integral & Sequence and Series of functions

Name of Teacher
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1. Examine the convergence of the improper integrals -

a) $\int_0^1 \frac{dx}{x^{1/2}(1-x)^{1/3}}$

b) $\int_0^\infty \frac{\sin x}{1+\sqrt{x}}\,dx$.

2. Show that $\int_0^1 \frac{dx}{(1-x^6)^{1/6}} = \pi/3$

3. Show that $\int_0^1 \frac{\log x}{x^n}\,dx$ is convergent if $n<1$ and divergent if $n \geq 1$.

4. Show that the sequence of functions $\{\frac{x^n}{n}\}$ defined on $[0,a]$ $(0<a<1)$ is uniformly convergent on $[0,a]$ and also find the limit functions.

5. Let $\{x_1, x_2, \ldots, x_n, \ldots\}$ be an enumeration of all rationals in $[0,1]$. Let

$$f_n(x) = 1, \quad x \in \{x_1, x_2, \ldots, x_n\}$$
$$= 0, \quad x \in [0,1] - \{x_1, x_2, \ldots, x_n\}$$

Show that the sequence $\{f_n\}$ is not uniformly convergent on $[0,1]$.

6. Show that the series $\sum_{n=1}^{\infty} \frac{n^5+1}{n^7+3}\left(\frac{x}{3}\right)^n$ is uniformly and absolutely convergent on $[0,1]$.

7. If $f_n(x) = \frac{nx}{1+n^2x^2} - \frac{(n-1)x}{1+(n-1)^2x^2}$, $x \in [0,1]$, show that $\int_0^1 \left(\sum_{n=1}^{\infty} f_n(x)\right)dx = \sum_{n=1}^{\infty}\left(\int_0^1 f_n(x)\,dx\right)$, although the series $\sum_{n=1}^{\infty} f_n(x)$ is not uniformly convergent on $[0,1]$.