AB

SEM 2
CC-3
UNIT-2 (SEQUENCE)

1. Give two examples of bounded ~~fin~~ sequences and two examples of unbounded sequences of real numbers (with explanation)

2. Give two examples of bounded above but unbounded below sequences of real numbers (with justification)

3. Give two examples of unbounded above but bounded below sequences of real numbers with proper justification.

4. Define limit of a sequence and prove that a sequence can have at most one limit.

5. Define convergent sequence. Give an example of a convergent sequence and a divergent sequence.

6. Using ϵ-δ definition prove that the sequence $\{\frac{1}{n}\}$ converges to 0.

7. Show that any constant sequence is convergent

8. Show that the sequence $\left\{\frac{n^2+1}{n^2}\right\}$ converges to 1.

9. Prove that a convergent sequence is bounded.

10. Give an example of a bounded sequence which is not convergent.

11. Give an example of a bounded sequence which is convergent.

12. Let $\{x_n\}$ and $\{y_n\}$ be two convergent sequences, with limit l and m resp, $m \neq 0$. Prove that

i) $\lim (x_n y_n) = lm$

ii) $\lim \left(\frac{x_n}{y_n}\right) = \frac{l}{m}, \; m \neq 0$

13. Let $\{x_n\}$ be a convergent sequence with limit l. Prove that the sequence $\{|x_n|\}$ converges to $|l|$.

14. State and prove ~~Sandwich~~ Sandwich theorem.

15. Prove that

$$\lim_{n\to\infty} \frac{3n^2+2n+1}{n^2+1} = 3$$

16. Prove that $\lim_{n\to\infty} \left(\sqrt{n+1}-\sqrt{n}\right) = 0$

17. Prove that

$$\lim_{n\to\infty} \left(\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \cdots + \frac{1}{\sqrt{n^2+n}}\right) = 1$$

18. Give the definitions of a divergent sequence diverges to ∞ and a divergent sequence diverges to $-\infty$. Give examples of each of the types.

19. a) Prove that $\lim_{n\to\infty} n^{\frac{1}{n}} = 1$

b) If $\lim x_n = 0$ and $a > 0$, then prove that $\lim a^{x_n} = 1$

c) ~~Prove~~ Prove that $\lim r^n = 0$ if $|r| < 1$

20. Let $\{x_n\}$ be a sequence of positive real numbers such that

$$\lim_{n\to\infty} \frac{x_{n+1}}{x_n} = l$$

Then show that if $0 \leq l < 1$, then $\lim x_n = 0$

21. Let $\{x_n\}$ be a sequence of positive real numbers such that $\lim_{n\to\infty} \sqrt[n]{x_n} = l$.

Prove that $\lim x_n = 0$ if $0 \leq l < 1$.

22. Let $\{x_n\}$ be a sequence defined by $x_n = \sqrt{2}$, $x_{n+1} = \sqrt{2x_n}$ for $n \geq 1$. Prove that $\lim x_n = 2$.

23. Prove that the seq^n $\{x_n\}$ defined by $x_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}$, $n \geq 1$ is m.i sequence.

24. Let $x_n = r^n$, $0 < r < 1$, $n \geq 1$. Show that $\{x_n\}$ is m.d seq^n.

25. State and prove monotone convergence theorems for m.i seq^n and m.d seq^n both.

26. State and prove Cantor's theorem on nested intervals.

27. Show that the limit of the seq^n $\left\{\left(1+\frac{1}{n}\right)^n\right\}$ is e.

28. Show that the seq^n $\{x_n\}$, where $x_n = 1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \cdots + \frac{1}{\lfloor n}$ is a m.i seq^n, bounded above and the $\lim x_n = e$.

29. Prove that the seq^n $\{x_n\}$ defined by $x_1 = \sqrt{2}$ and $x_{n+1} = \sqrt{2x_n}$, $n \geq 1$ converges to 2.

30. Prove that the seq^n $\{x_n\}$ defined by $x_1 = \sqrt{7}$ and $x_{n+1} = \sqrt{7+x_n}$, $\forall n \geq 1$ converges to the +ve root of the equ^n $x^2 - x - 7 = 0$