Assingment for SEM-IV

C.C-10, unit-1

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1. State and prove Lami's Theorem. Is the converse of Lami's Th is true?

2. State and Varignon's Theorem of moments.

3. If the two like parallel forces P and Q acting on a rigid body at A & B be interchanged in position show that the point of application of the resultant will be displaced along AB through a distance d where $d = \frac{P-Q}{P+Q} AB$. $(P > Q)$

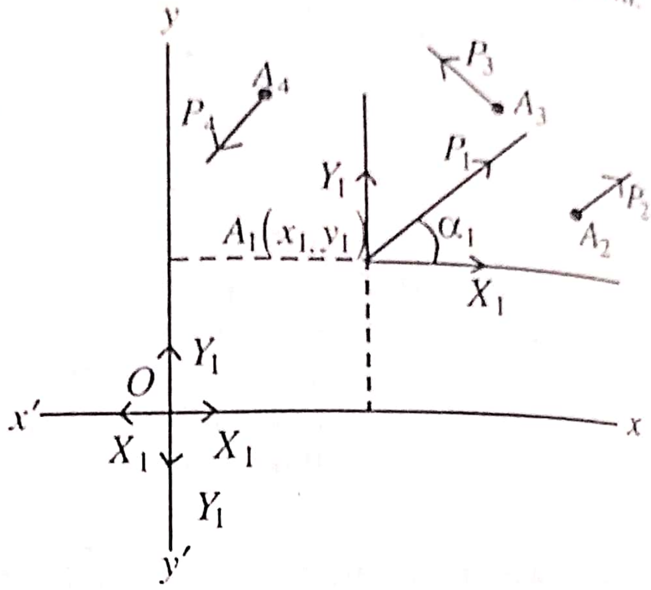
4. Forces P, Q, R acting along OA, OB, OC are in equilibrium. If O be the circumcentre of the triange ABC, prove that

$$\frac{P}{\frac{1}{b^2}+\frac{1}{c^2}-\frac{a^2}{b^2c^2}} = \frac{Q}{\frac{1}{c^2}+\frac{1}{a^2}-\frac{b^2}{c^2a^2}} = \frac{R}{\frac{1}{a^2}+\frac{1}{b^2}-\frac{c^2}{a^2b^2}}$$

where a, b, c are the lengths of the side BC, CA, AB.

Theorem. *Any system of coplanar forces acting on a rigid body can be reduced to a single resultant force acting at any arbitrary point in the plane, together with a single resultant couple of moment which is equal to the algebraic sum of moments of the given forces about that arbitrary point.*

Let $x'Ox$ and $y'Oy$ be two rectangular axes with O as origin in the plane of action of the forces. Let $P_1, P_2, \cdots$ be the given coplanar forces acting at the different points $A_1(x_1, y_1), A_2(x_2, y_2), \cdots$.



Let $(X_1, Y_1), (X_2, Y_2), \cdots$ be the components parallel to co-ordinate axes and $\alpha_1, \alpha_2, \cdots$ inclination of directions of those forces with Ox. Therefore

$$X_1 = P_1 \cos\alpha_1 \text{ and } Y_1 = P_1 \cos(90^\circ - \alpha) = P_1 \sin\alpha .$$

Now we introduce, at O, a pair of equal and opposite forces X_1, X_1 acting in the line $x'Ox$ and a pair of equal and opposite forces Y_1, Y_1 acting in the line $y'Oy$.

Now force P_1 at $A_1 \equiv$ force X_1 at A_1 and force Y_1 at A_1.

And force X_1 at $A_1 \equiv$ a force X_1 at O along Ox and a couple of moment $-(y_1 X_1)$.

And force Y_1 at $A_1 \equiv$ a force Y_1 at O along Oy and a couple of moment $+(x_1 Y_1)$.

Again these two couples can be compounded into a single couple of moment $= x_1 Y_1 - y_1 X_1$.

Thus the force P_1 at A_1 is equivalent ot a force X_1 at O along Ox, a force Y_1 at O along Oy and a couple of moment $x_1 Y_1 - y_1 X_1$.

Hence for the given system we have :

(i) forces $X_1, X_2, \cdots$ at O along Ox,

(ii) forces $Y_1, Y_2, \cdots$ at O along Oy,

and (iii) couples of moments $(x_1 Y_1 - y_1 X_1), (x_2 Y_2 - y_2 X_2), \cdots$.

Combining all the components along Ox and Oy separately and combining all the couples we get

X = sum of the components along Ox

$$= X_1 + X_2 + \cdots = \sum X_i = \sum P_i \cos\alpha_i$$

Y = sum of the components along Oy

$$= Y_1 + Y_2 + \cdots = \sum Y_i = \sum P_i \sin\alpha_i$$

and G = sum of the couples of moments about $O = \sum (x_i Y_i - y_i X_i)$.

Hence given system has been reduced to a single force $X = \sum X_i$ along Ox, a single force $Y = \sum Y_i$ along Oy and a couple $G = \sum (x_i Y_i - y_i X_i)$ about O.

The forces X, Y at O reduce to a single resultant force, say R, acting at O in a direction θ with Ox.

Then, $R\cos\theta = X = \sum X_i$

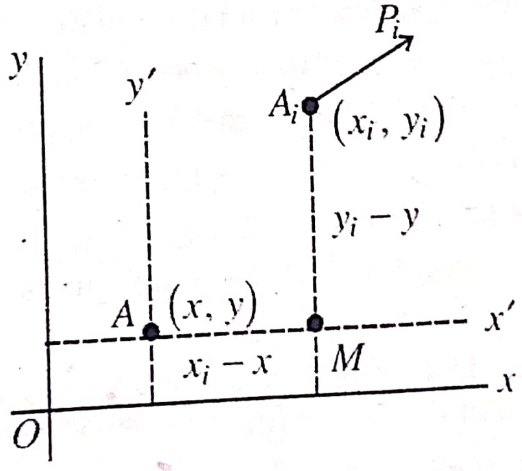
$R\sin\theta = Y = \sum Y_i$

$$\Rightarrow R = \sqrt{X^2 + Y^2} \text{ and } \tan\theta = \frac{Y}{X}.$$

Therefore, the given system reduces to a single force R at O together with a single couple G.

2.2. Moment about any point as base

Through O let us take any pair of rectangular axes Ox and Oy. Let a given system of forces be reduced, w.r.t. a base O, as origin, to single force R, having components $X = \sum X_i$ and $Y = \sum Y_i$ along the axes, together with a couple $G = \sum (x_i Y_i - y_i X_i)$.



Let $A(x, y)$ be any other point chosen as base and corresponding moment G'. Then

$$G' = \sum (x_i' Y_i - y_i' X_i)$$

$= \sum \{(x_i - x)Y_i - (y_i - y)X_i\}$ where $x_i' = x_i - x,\ y_i' = y_i - y$

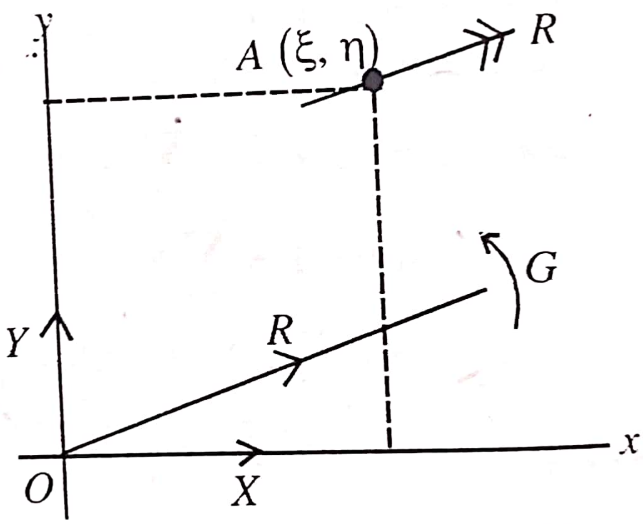
$= \sum (x_i Y_i - y_i X_i) - \sum x Y_i + \sum y X_i$

$= G - x\sum Y_i + y\sum X_i$ [$\because$ x and y are same for each point on the plane of forces]

$= G - xY + yX$.

Note : If $R = 0$ i.e., $X = 0$ and $Y = 0$ i.e., if the given system is equivalent to a couple then $G' = G$. Hence if the given system of forces reduces to a couple only, which in this case will be same, whatever point is chosen as origin.

2.3. Equation of the line of action of Resultant



Let O be the origin (base) in the plane of given forces and Ox, Oy are rectangular axes. Let the given system of forces be reduced to a single force R acting at O, having the components $X = \sum X_i$ and $Y = \sum Y_i$ along the axes, together with a couple $G = \sum (x_i Y_i - y_i X_i)$.

We know that if $R \neq 0,\ G \neq 0$, then also, the single force and single couple can be combined into a single resultant force, same in magnitude and direction as R at O, but shifted in position. So, to find the equation of the line of action R, we shift the point of application O to any point A is the plane of forces with direction as R at O.

Let (ξ, η) be the co-ordinates of any point A on the line of action of R. Then the algebraic sum of the moments of the given forces about this point $A(\xi, \eta)$ is zero, i.e., $G' = 0$.

So, $G' = G - \xi Y + \eta X$ gives

$$G - \xi Y + \eta X = 0.$$

Hence the locus of the point (ξ, η) is $G - xY + yX = 0$ which is the also equation of the line of action of the resultant R.

The moments of a system of forces about the points $(0, 0)$, $(a, 0)$, $(0, a)$ *are* aw, $2aw$, $3aw$ *respectively. Find the components of their resultant parallel to the co-ordinate axes and the equation to its line of action.*

Ans. Let the system of forces acting on a body be reduced to the forces X, Y along the co-ordinate axes and a couple of moment G.

We know that moment about any point (x, y) is given by

$$G' = G - xY + yX .$$

So, taking moment about $(0, 0)$, $(a, 0)$ and $(0, a)$ then

we have $$aw = G - 0.Y + 0.X$$

$$2aw = G - aY + 0X$$

and $3aw = G - 0Y + aX$

which gives $X = 2w$, $Y = -w$ and $G = aw$.

Again equation of the line of action is given by

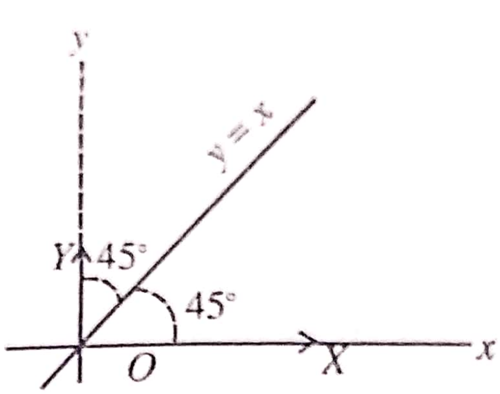
$$G - xY + yX = 0 .$$

So, in this problem, the equation of the line of action is

$aw - x(-w) + y(2w) = 0$

i.e., $x + 2y + a = 0$.

A system of coplanar forces has the total moments H, 2H respectively about points whose co-ordinates are $(2a, 0), (0, a)$ referred to fixed rectangular axes. The total resolved parts of the forces along the line $y = x$ vanishes. Find the points in which the line of action of the resultant meets the co-ordinate axes.



Ans. Let the system of coplanar forces acting on a rigid body be reduced to the forces *X*, *Y* along the co-ordinate axes *Ox*, *Oy* and a couple *G*.

Now moment about any point (x, y) is given by $G' = G - xY + yX$.

So, when $x = 2a, y = 0$ then $G' = H$ and

when $x = 0, y = a$ then $G' = 2H$.

Therefore, $H = G - 2aY$ (1)

and $2H = G + aX$. (2)

Given that total resolved parts of the forces *X*, *Y* along the line $y = x$ vanishes. So,

$X\cos 45° + Y\cos 45° = 0$

i.e., $X + Y = 0$. (3)

Solving (1), (2), and (3) we get

$$X = -\frac{H}{a}, \quad Y = \frac{H}{a} \text{ and } G = 3H.$$

Now the equation of the line of action of the resultant is

$G - xY + yX = 0$

i.e., $3H - x\dfrac{H}{a} + y\left(\dfrac{-H}{a}\right) = 0$

i.e., $\dfrac{x}{3a} + \dfrac{y}{3a} = 1$.

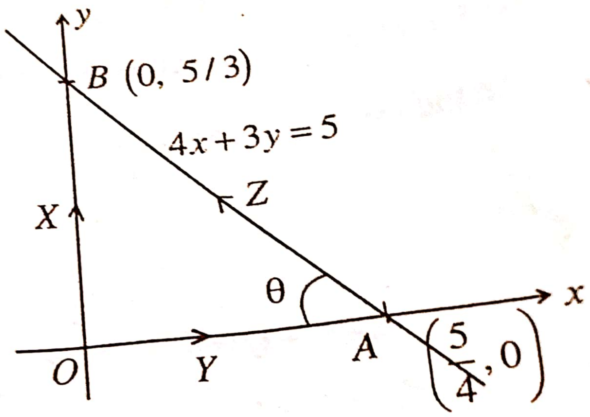
which shows that the line of action of the resultant meets the co-ordinate axes at $(3a, 0)$ and $(0, 3a)$.

Ex. 10. *The straight line* $4x+3y=5$ *meets the rectangular axes* Ox, Oy *at A and B respectively. If the forces X, Y, Z act along the lines OB, OA and AB find the magnitude of the resultant and the equation of the line of action.*

Ans. *Hint.* The lines of action of the forces are shown in the adjacent figure.

Let $\angle OAB = \theta$. then

$$\tan\theta = \frac{OB}{OA} = \frac{5}{3}\Big/\frac{5}{4} = \frac{4}{3}.$$



Let X', Y' be the sum of the resolved parts of the forces along Ox and Oy respectively. So,

$$X' = Y - Z\cos\theta = Y - Z\cdot\frac{3}{5}$$

$$Y' = X + Z\sin\theta = X + Z\cdot\frac{4}{5}$$

$\therefore$ Resultant $R = \sqrt{X'^2 + Y'^2} = \sqrt{\left(Y - \frac{3Z}{5}\right)^2 + \left(X + \frac{4Z}{5}\right)^2} = \sqrt{X^2 + Y^2 + Z^2 - \frac{6}{5}YZ + \frac{8}{5}XZ}$

$G =$ sum of the moments about O.

$$= X\cdot O + Y\cdot O + Z\cdot\frac{5}{\sqrt{4^2 + 3^2}} = \frac{5Z}{5} = Z.$$

So, the equation of the line of action of the resultant is

$$G - xY' + yX' = 0$$

$$Z - x\left(X + \frac{4Z}{5}\right) + y\left(Y - \frac{3Z}{5}\right) = 0$$

$$5(xX - yY) + Z(4x + 3y - 5) = 0.$$

4.1. Introduction

We know smooth bodies to be bodies such that if they be in contact, the only action between them is perpendicular to both surfaces at the point of contact. With smooth bodies, therefore, there is no force tending to prevent one body sliding over the other. If a perfectly smooth body be placed on a prefectly smooth inclined plane, there is no action between the plane and the body to prevent the latter from sliding down the plane, hence the body will not remain at rest on the plane unless some external force be applied to it.

Practically, however, there are no bodies which are perfectly smooth; there is always some force between two bodies in contact to prevent one sliding upon the other. Such a force is called the force of friction.

Before going to definition at first we define Passive forces.

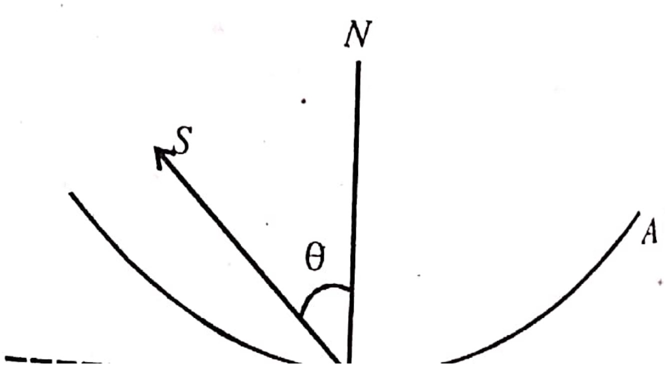
Passive Forces : In some investigations we come across a type of forces which may be described as *passive forces*. Such forces come into operation because of other forces and their magnitudes and directions are generally determined by the requirements of eqilibrium . They are variable in magnitude and direction and they will act with such intensity and in such a direction as to preserve equilibrium *if they can*.

Examples: (i) The force of reaction between a curve and a particle constrained to move on it,

(ii) The resistance of air or any other fluid to a body moving through it,

(iii) the tension in a rope tethering an animal to a stake .

Consider two bodies A and B in contact with each other at a point O. The normal ON at O to the common tangent plane to the surfaces in generally called the common normal . Let S be the mutual reaction between the two bodies and let it make an ...



Definition : *If two bodies be in contact with one another , the property of the two bodies , by virtue of which a force is exerted between them at their point of contact to prevent one body sliding on other, is called friction ; also the force exerted is called the force of friction.* It is clear that friction is a passive force. The mutual reaction between two bodies is a passive force and exists only because of other forces acting on the bodies. Friction, being a component of the mutual reaction, is a passive force.

4.5. Coefficient of Friction

The constant ratio of the limiting friction to the normal pressure is called the coefficient of friction, and is generally denoted by μ; hence, if F be the limiting friction, and R the normal pressure, then

$$\frac{F}{R} = \mu \quad \text{or} \quad F = \mu R \,.$$

The values of μ are widely different for different pairs of substances in contact; no pairs of substances are, however, known for which the coefficient of friction is so great as unity, i.e., in no case μ has been found to be greater than unity.

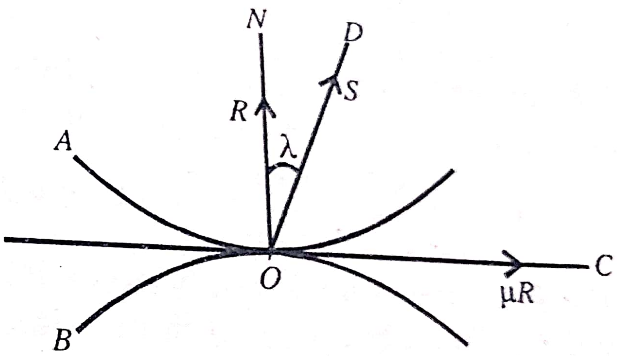
Angle of Friction : *When the equilibrium is limiting, if the friction and the normal reaction be compounded into one single force, the angle which this force makes with the normal is called the angle of friction*, and the single force is called the *resultant reaction.*

In the figure representing two bodies A and B in contact at O. Let ON and OC be the direction of the normal force R and friction μR. Let OD be the direction of the resultant reaction S, so that the angle of friction is $\angle NOD = \lambda$ (say). Then

$$S\cos\lambda = R \text{ and } S\sin\lambda = \mu R$$

$$\Rightarrow \tan\lambda = \mu$$

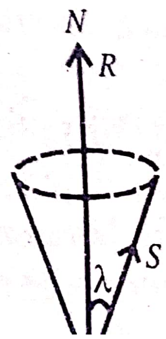
and $S = R\sqrt{1+\mu^2}$.



Hence we see that *the coefficient of friction is equal to the tangent of the angle of friction.*

Since the greatest value of the friction is μR, it follows that the greatest angle which the direction of resultant reaction can make the normal is $\lambda = \tan^{-1}(\mu)$.

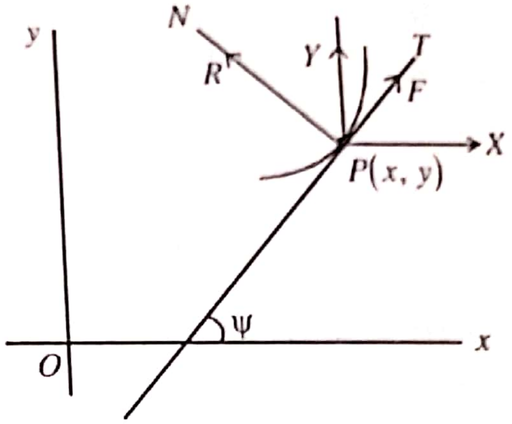
Cone of Friction : *If with the point of contact O as vertex, the common normal as axis and the angle of friction* $\lambda = \tan^{-1}(\mu)$ *as the semi-vertical angle a cone is described, that cone is called the cone of friction.*



ow we shall discuss the condition of equilibrium of a body resting on her body under the action of external forces.

.8. Theorem of Equilibrium on a rough Curve

Equilibrium of a particle constrained to rest on a rough curve under any given forces.



Let the particle be rest at the point $P(x, y)$ on the plane curve and X, Y the component forces parallel to the axes of co-ordinates.

If R be the normal reaction and F the friction along the tangent PT which makes an angle ψ with the axis of x. Resolving along PT and perpendicular to PT, we get

$$X\cos\psi + Y\sin\psi + F = 0$$

and $$-X\sin\psi + Y\cos\psi + R = 0$$

Which $\Rightarrow$ $$F = -(X\cos\psi + Y\sin\psi)$$

and $$R = (X\sin\psi - Y\cos\psi)$$

If μ be the coefficient of friction , for eqilibrium we must have F numerically less than or equal to μR . So,

$$|F| \le \mu |R|$$

i.e., $$|X\cos\psi + Y\sin\psi| \le \mu |X\sin\psi - Y\cos\psi|$$

i.e., $$\left(\frac{X + Y\tan\psi}{X\tan\psi - Y}\right)^2 \le \mu^2$$

For the plane curve we have $\tan\psi = \dfrac{dy}{dx}$ at P . Therefore,

$$\left(X + Y\frac{dy}{dx}\right)^2 \le \mu^2\left(X\frac{dy}{dx} - Y\right)^2 \quad \ldots \quad (1)$$

Equality sign holds good in the position of limiting equlibrium .

Cor. If the equation of the rough plane curve be $f(x, y) = 0$ then

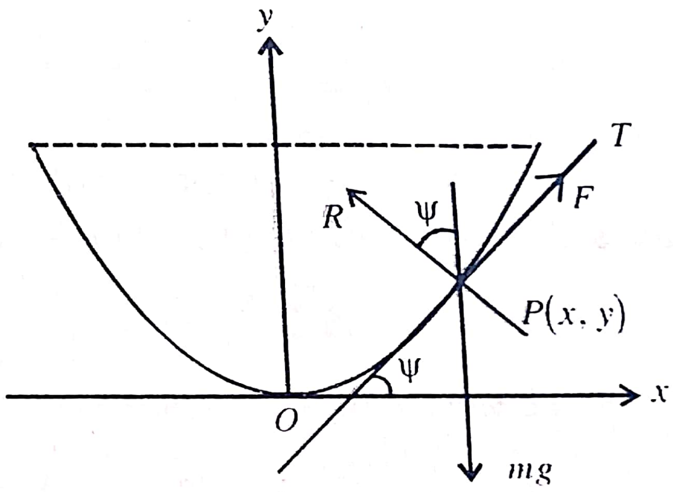
we have $$\frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = 0$$

i.e., $$\frac{dy}{dx} = -\frac{f_x}{f_y} \quad \text{where } f_x = \frac{\partial f}{\partial x},\ f_y = \frac{\partial f}{\partial y}$$

Ex. 2. *A cycloid is placed with its axis vertical and vertex downwards. Show that a particle cannot rest at any point of the curve which is higher than* $2a\sin^2\lambda$ *above its lowest point where* λ *is the angle of friction and a is the radius of the generating circle of the cycloid.* [V.H. 2004,2003]

Ans. The parametric eqation of the cycloid is

$$\left.\begin{aligned} x &= a(\theta + \sin\theta) \\ y &= a(1-\cos\theta) \end{aligned}\right\} \quad \cdots \quad (1)$$



Let $P\ (x,y)$ be the position of rest of the particle of mass m on the cycloid and let the tangent PT make an angle ψ with Ox. So, at P,

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$$\tan\psi = \frac{dy}{dx} = \frac{a\sin\theta}{a(1+\cos\theta)} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} = \tan\frac{\theta}{2}$$

$$\therefore \quad \psi = \frac{\theta}{2} \qquad \ldots \quad (2)$$

For equilibrium , the resultant reaction at P must balances the weight mg of the particle. Hence, for equilibrium , $\psi \le \lambda$ ... (3)

Now height of P above $Ox = y = a(1-\cos\theta)$

$$= 2a\sin^2\frac{\theta}{2}$$

$$= 2a\sin^2\psi \qquad \text{[by (2)]}$$

Thus equilibrium to be possible ,we must have

$$y = 2a\sin^2\psi \le 2a\sin^2\lambda \qquad \text{[by (3)]}$$

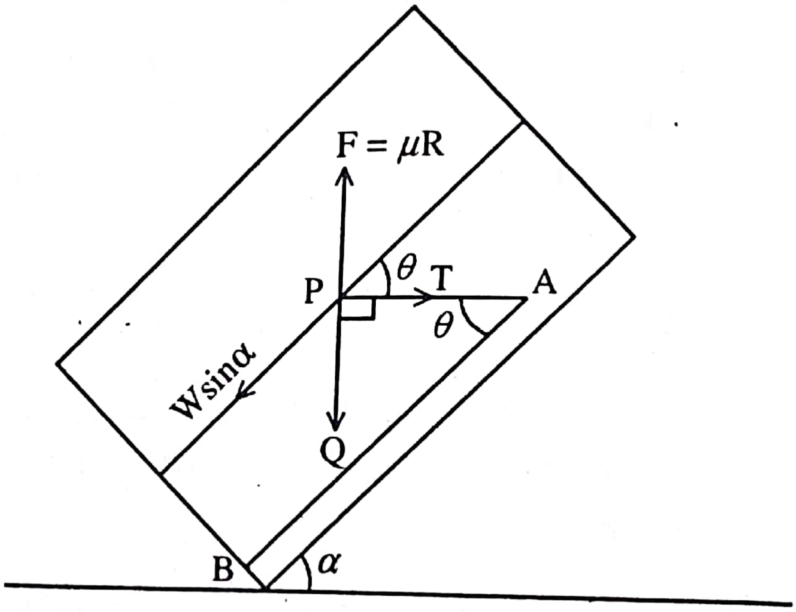
Hence the particle cannot rest at a point whose height above O is greater than $2a\sin^2\lambda$.

A heavy particle is placed on a rough plane inclined at an angle α to the horizon, and is connected by a stretched weight less string AP to a fixed point A in the plane. If AB be the line of greatest slope and θ the angle PAB when the particle is on the point of slipping, show that

$$\sin\theta = \mu \cot\alpha.$$

Interpret the result when $\mu \cot \alpha$ is greater than unity.

Solution : Since the particle is attached to the fixed point A by an inelastic string, the particle can only move so as to describe a circle about A of radius AP.



∴ The direction of motion at P being along the tangent to this circle will be ⊥ to AP, in the direction PQ.

∴ The limiting friction $\mu W \cos\alpha$ acts along QP.

Applying lami's theorem to the 3 forces in equilibrium we get,

$$\frac{W\sin\alpha}{\sin 90^\circ} = \frac{\mu W\cos\alpha}{\sin(180^\circ - \theta)} = \frac{T}{\sin(90^\circ + \theta)}$$

$W \sin\alpha \sin\theta = \mu W \cos\alpha$ [using the first equality].

$$\text{or, } \sin\theta = \frac{\mu\cos\alpha}{\sin\alpha} \qquad \text{or, } \sin\theta = \mu\cot\alpha. \quad \text{... (i)}$$

If $\mu\cot\alpha$ is numerically > unity, then we see from (i) that there is no real value for θ. Hence there is no real position of limiting

Using above result in (1) we get

$$\left(X - Y\frac{f_x}{f_y}\right)^2 \le \mu^2\left(-X\frac{f_x}{f_y} - Y\right)^2$$

or, $$\frac{(Xf_y - Yf_x)^2}{(Xf_x + Yf_y)^2} \le \mu^2$$

or, $$\frac{(X^2+Y^2)(f_y^2+f_x^2)}{(Xf_x+Yf_y)^2} \le 1+\mu^2$$

or, $$\frac{1}{1+\mu^2} \le \frac{(X f_x + Y f_y)^2}{(X^2+Y^2)(f_x^2+f_y^2)} \quad \ldots \quad (2)$$

If λ be the angle of friction then $\mu = \tan\lambda$. Therefore,

$$\cos^2\lambda \le \frac{(X f_x + Y f_y)^2}{(X^2+Y^2)(f_x^2+f_y^2)}$$

or, $$\cos\lambda \le \frac{(X f_x + Y f_y)}{\sqrt{X^2+Y^2}\,\sqrt{f_x^2+f_y^2}}$$

Cor. For rough surface $f(x,y,z)=0$ we get the direction cosines of the normal to the surface at the point (x,y,z) are proportional to f_x, f_y, f_z.

If X, Y, Z be the components of the given forces parallel to the co-ordinate axes at that point then

$$\cos\theta = \pm\frac{X f_x + Y f_y + Z f_z}{\sqrt{X^2+Y^2+Z^2}\,\sqrt{f_x^2+f_y^2+f_z^2}} = \pm\frac{\sum X f_x}{\sqrt{\sum X^2}\sqrt{\sum f_x^2}}$$

where θ being the angle between the resultant with normal. But in equilibrium

$$\theta \le \lambda$$

i.e., $\tan\theta \le \tan\lambda = \mu$

i.e., $\sec^2\theta = 1+\tan^2\theta \le 1+\mu^2$

i.e., $$\cos^2\theta \ge \frac{1}{1+\mu^2}$$

i.e., $$\left(\sum X f_x\right)^2 \ge \frac{1}{1+\mu^2}\sum X^2 \sum f_x^2$$

...id. ...s of the

A particle rests on the surface $xyz = c^2$ under the action of a constant force parallel to the axis of z; show that the curve of intersection of the surface with the cone $\frac{1}{x^2} + \frac{1}{y^2} = \frac{\mu^2}{z^2}$ will separate the part of the surface on which equilibrium is possible from that on which it is not possible.

Solution : $\phi(x, y, z) = xyz - c^2 = 0$.

The external force is given by

$X = 0,\ Y = 0,\ Z = K$, where K is a constant.

Then condition for equilibrium for a particle in equilibrium on a rough surface $\phi(x, y, z) = 0$ where co-eff. of friction is μ under the given external force components X, Y, Z is given by

$$\frac{\left(X\phi_x + Y\phi_y + Z\phi_z\right)^2}{\left(X^2+Y^2+Z^2\right)\left(\phi_x^2+\phi_y^2+\phi_z^2\right)} \geq \frac{1}{1+\mu^2} \quad \text{... (ii)}$$

Hence $\phi_x = \frac{\partial \phi}{\partial x} = yz \qquad \phi_y = \frac{\partial \phi}{\partial y} = zx \qquad \phi_z = \frac{\partial \phi}{\partial z} = xy.$

Substituting the condition in (ii) we get

$$\frac{(K\,xy)^2}{K^2(y^2z^2+z^2x^2+x^2y^2)} \geq \frac{1}{1+\mu^2} \quad \textit{i.e.} \quad \frac{x^2y^2+y^2z^2+z^2x^2}{x^2y^2} \leq 1+\mu^2$$

$$\text{or, } 1+\frac{z^2}{y^2}+\frac{z^2}{x^2} \leq 1+\mu^2 \quad \text{or, } \frac{1}{y^2}+\frac{1}{x^2} \leq \frac{\mu^2}{z^2}$$

$$\textit{i.e.} \quad \frac{1}{x^2}+\frac{1}{y^2} \leq \frac{\mu^2}{z^2}. \quad \text{... (iii)}$$

Hence the curve of intersection of the surface given by

$$\frac{1}{x^2}+\frac{1}{y^2}=\frac{\mu^2}{z^2} \quad \text{... (iv)}$$

with the given surface (i) separates the parts of the latter surface on which equilibrium is possible. [*i.e.* condition (ii) holds] from those parts on which it is not [*i.e.* where $\frac{1}{x^2}+\frac{1}{y^2} > \frac{\mu^2}{z^2}$].

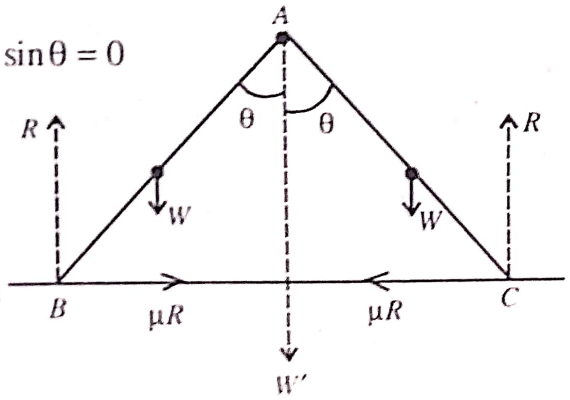
Ex. 11. *Two equal ladders of weight W are placed so as to lean aganist each other at an angle 2θ, with their ends resting on a rough horizontal floor, the coefficient of friction of which w.r.t.either being μ, where $\tan\theta > \mu > \frac{1}{2}\tan\theta$.*

If W' be the weight which placed on the top causes the ladders to slip, show that $W' = W\dfrac{2\mu - \tan\theta}{\tan\theta - \mu}$. *Explain the case when* $\mu < \frac{1}{2}\tan\theta$ *or* $\mu > \tan\theta$.

Ans. Let AB and AC be two equal ladders, each of weight W meet at A. $\angle BAC = 2\theta$. Let R be the normal reactions at B and C then friction at B and C are μR along BC and CB respectively. Considering the equilibrium of the whole system with weight W' at A, we get

$$2R = 2W + W' \qquad \cdots \qquad (1)$$

Again considering the equilibrium of the ladder AB only, taking moments about A, we get



$$-R\cdot AB\sin\theta + \mu R\cdot AB\cos\theta + W\frac{AB}{2}\sin\theta = 0$$

or, $R\sin\theta = \mu R\cos\theta + \dfrac{W}{2}\sin\theta$

or, $\left(R - \dfrac{W}{2}\right)\tan\theta = \mu R$

or, $(2W + W' - W)\tan\theta = \mu(2W + W')$ [by (1)] or, $W'(\tan\theta - \mu) = W(2\mu - \tan\theta)$

$$\therefore\ W' = W\left(\frac{2\mu - \tan\theta}{\tan\theta - \mu}\right) \text{ for } \tan\theta > \mu > \frac{1}{2}\tan\theta .$$

If $\mu < \frac{1}{2}\tan\theta$ or $\mu > \tan\theta$ in each case, the value of W' is negative, showing that instead of placing a weight at A, an upward force must be applied at A in order that the ladders may be in limiting equilibrium under the circumstances.

or, $W'(\tan\theta-\mu)=W(2\mu-\tan\theta)$

$$\therefore W'=W\left(\frac{2\mu-\tan\theta}{\tan\theta-\mu}\right) \text{ for } \tan\theta>\mu>\frac{1}{2}\tan\theta .$$

If $\mu<\frac{1}{2}\tan\theta$ or $\mu>\tan\theta$ in each case, the value of W' is negative, showing that instead of placing a weight at A, an upward force must be applied at A in order that the ladders may be in limiting equilibrium under the circumstances.