Solve the problems :

① Let the two propositions be

p: It is cold

and q: It is raining

Write the statement against the following symbol.

i) $q \wedge \sim p$ (ii) $p \vee q$ (iii) $\sim(p \vee q)$

② Let p : Today is Friday

q : It is raining

r : It is hot.

Write the statement against the following symbol.

i) $\sim q \rightarrow (r \wedge p)$

(ii) $(p \vee q) \leftrightarrow r$

(iii) $(p \wedge \sim q) \rightarrow \sim r$

③ Construct the truth tables for the following statements :—

a) $(p \wedge q) \rightarrow (p \uparrow q)$

b) $(\sim p \uparrow q) \rightarrow (p \downarrow \sim q)$

c) $(\sim(p \uparrow q) \wedge (p \downarrow q)) \rightarrow (p \downarrow (q \uparrow p))$

Some simple fundamental problems

(1) Write the following statement in symbolic form:—

"If either Neha takes Mathematics or Raja takes Physics, then Suvo takes computer"

Solⁿ: Let p : Neha takes Mathematics
q : Raja takes Physics
r : Suvo takes computer

∴ The given statement can be written in symbolic forms as: $(p \vee q) \rightarrow r$

~~Solve the problems :-~~

(2) Using the statements
p : Neha is Rich
q : Neha is happy

Write the following statements in a symbolic form
(i) Neha is poor but happy
(ii) Neha is rich or unhappy
(iii) Neha is neither rich nor happy
(iv) Neha is poor or she is both rich and unhappy.

Solⁿ: (i) $\sim p \wedge q$
(ii) $p \vee \sim q$
(iii) $\sim p \vee \sim q$
(iv) $\sim p \vee (p \wedge \sim q)$

(4) Without using truth table prove that

$$\sim p \wedge (\sim q \wedge r) \vee (q \wedge r) \vee (p \wedge r) \equiv r$$

Soln: $\sim p \wedge (\sim q \wedge r) \vee (q \wedge r) \vee (p \wedge r)$

$\equiv [\sim p \wedge (\sim q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)]$ [Asso law]

$\equiv [(\sim p \wedge \sim q) \wedge r] \vee [(q \vee p) \wedge r]$ [Dist Law]

$\equiv [(\sim p \wedge \sim q) \vee (q \vee p)] \wedge r$ [Dist. law]

$\equiv [\sim (p \vee q) \vee (q \vee p)] \wedge r$ [De Morgan's law]

$\equiv [\sim (p \vee q) \vee (p \vee q)] \wedge r$ [Comm. Law]

$\equiv T \wedge r$ [Negation law]

$\equiv r$ [Identity law]

Exe Prove that (without using truth table)

1. $p \leftrightarrow q \equiv (p \vee q) \rightarrow (p \wedge q)$, p, q are propositional variables

2. $[p \wedge (\sim p \vee q)] \vee [q \wedge \sim (p \wedge q)] \equiv q$

3. $[\sim p \wedge (\sim q \wedge r)] \vee (q \wedge r) \vee (p \wedge r) \equiv r$

4. $p \rightarrow (q \rightarrow r) \equiv p \rightarrow (\sim q \vee r) \equiv (p \wedge q) \rightarrow r$

Tautologies

~~1. Without using truth tables prove that~~

(1) Without using truth table prove that

$$p \vee (p \wedge q) \equiv p,$$

where p, q are propositional variables.

Soln.

$$p \vee (p \wedge q) \equiv (p \wedge T) \vee (p \wedge q) \quad [\because p \wedge T \equiv p]$$
$$\equiv p \wedge (T \vee q) \quad [\text{Distributive law}]$$
$$\equiv p \wedge T \quad [\because T \vee q \equiv T]$$
$$\equiv p \quad [\because p \wedge T \equiv p]$$

$\therefore \; p \vee (p \wedge q) \equiv p$

(2) Prove that $\sim(p \vee q) \vee (\sim p \wedge q) \equiv \sim p$, where p, q are propositional variables (using using truth table)

$$\sim(p \vee q) \vee (\sim p \wedge q) \equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q) \quad [\text{De Morgan's law}]$$
$$\equiv \sim p \wedge (\sim q \wedge q) \quad [\text{Dist. law}]$$
$$\equiv \sim p \wedge T$$
$$\equiv \sim p \quad [\text{Proved}]$$

(3) Prove that without using truth table,

$$p \rightarrow (q \rightarrow r) \equiv (p \wedge q) \rightarrow r,$$ where p, q, r are propositional variables

$$p \rightarrow (q \rightarrow r) \equiv p \rightarrow (\sim q \vee r) \quad [\text{conditional equivalence}]$$
$$\equiv \sim p \vee (\sim q \vee r)$$
$$\equiv (\sim p \vee \sim q) \vee r \quad [\text{Associative law}]$$
$$\equiv \sim(p \wedge q) \vee r \quad [\text{De Morgan's law}]$$
$$\equiv (p \wedge q) \rightarrow r \quad [\text{Proved}]$$