Chapter 2

Map Projectio[n]

The term *projection* is commonly used to designate the phenomena of image production of an object onto a surface or plane. This involves the application of a set of principles, procedures and purposes. Broadly speaking, map projection is defined as the systematic drawing of a network of parallels and meridians on a plain sheet of paper portraying a part or whole of the earth's surface. Naturally, it is scale-dependent and is done in accordance with a set of geometric and mathematical principles to satisfy certain objectives of the user.

Map projection is a device by which the curved surface of the earth is represented on a flat plane. The operational process essentially involves dimensional transformation, i.e., a 2-dimensional representation of the 3-dimensional figure of the earth. This produces deformations which are inevitable because the surface of the generating globe and the surface or plane of projection are not *geometrically applicable.*

Mathematically, the general equations describing such transformation in map projection are:

$$u = f_1 (\lambda, \phi) \quad \text{... (i)}$$

$$v = f_2 (\lambda, \phi) \quad \text{... (ii)}$$

where, λ, ϕ define the coordinates of positions on the original 3-dimensional surface, u, v describe the corresponding coordinates on the transformed 2-dimensional plane and f_1 and f_2 are real, single-valued, continuous and differentiable functions of λ and ϕ in certain domains so that the Jacobian determinant does not vanish:

$$J = \begin{vmatrix} \frac{\partial u}{\partial \lambda}, \frac{\partial u}{\partial \phi} \\ \frac{\partial v}{\partial \lambda}, \frac{\partial v}{\partial \phi} \end{vmatrix} \neq 0$$

On the transformed plane, the rectang[ular] coordinates (x, y) and polar coordinates (ρ, A) [of] the same point with geographical coordina[tes] (λ, ϕ) or spherical coordinates (θ, r) are given b[y]

$$x = f_1 (\lambda, \phi) \qquad \rho = f_1 (\theta, r)$$
$$y = f_2 (\lambda, \phi) \quad \text{or} \quad A = f_2 (\theta, r)$$

Therefore, x and y or ρ and A are specif[ic] functions of latitude and longitude and one singl[e] point (λ, ϕ) or (θ, r) on the earth is represented b[y] one and the only point (x, y) or (ρ, A) on the ma[p]. Thus, map projection is *reversible* and *unique.*

Scale Factor

Map projection is a 2-step process in which the earth is first reduced to a generating globe of [a] desired size and then the generating globe is projected onto a plane. The transformation of a globe to a plane is identical to the problem of trying to make the skin of an orange exactly coplaner and coincident with a table top without contortions, stretching and even tearing. Hence deformation and distortions are inevitable in map projection.

The scale in which the generating globe (a 3-dimensional figure) is conceptualised is called the *principal scale.* On maps it is correctly maintained only at selected points or lines (i.e., the point of tangency or the lines of contact of the projection plane or developable surface with the generating globe). Elsewhere on the map where distortions occur, the principal scale becomes significantly different from that in which the map is actually generated. The scale of the resultant map is termed as the *real scale.* It is the differential stretching and contortions of the generating globe that make the real scale unequal at each and every point on the map. Hence, on the resultant map, a one-to-one correspondence for all points is a practical impossibility. The ratio between the principal scale and the real scale at any point on the map is called the *scale factor* at that point. Mathematically speaking,

Scale Factor,

$$SF = \frac{\text{Denominator of the Principal Scale}}{\text{Denominator of the Real Scale}}$$

Radial Scale Factor and Tangential Scale Factor

Tissot's (1850) *law of deformation* states that '... at each point of the spherical surface there exists at least two perpendicular directions which reappear at right angles to each other on the projection, although all other angles at that point may be altered from their original disposition.' On a map these two directions are as follows—one along a parallel and the other along a meridian. The scale factor measured along a parallel is called the *parallel scale factor* or *tangential scale factor* while that measured along a meridian is called the *meridional scale factor* or *radial scale factor*. The equations for derivation are:

Tangential Scale Factor (TSF)

$$= \frac{\left(\begin{array}{c}\text{Denominator of the Principal Scale}\\ \text{along a parallel } (\phi)\end{array}\right)}{\left(\begin{array}{c}\text{Denominator of the Real Scale}\\ \text{along the same parallel } (\phi)\end{array}\right)}$$

$$= \frac{\text{Length of a parallel on globe } (L_{\phi g})}{\text{Length of the same parallel on map } (L_{\phi m})}$$

Hence, along a parallel ϕ, tangential scale factor,

$$TSF = \frac{L_{\phi g}}{L_{\phi m}} \quad \text{... (i)}$$

and tangential scale is expressed by

$$1 : TSF \quad \text{... (ii)}$$

Radial Scale Factor (RSF)

$$= \frac{\left(\begin{array}{c}\text{Denominator of the Principal Scale}\\ \text{along a meridian } (\lambda)\end{array}\right)}{\left(\begin{array}{c}\text{Denominator of the Real Scale}\\ \text{along the same meridian } (\lambda)\end{array}\right)}$$

$$= \frac{\text{Length of a meridian on globe } (L_{\lambda g})}{\text{Length of the same meridian on map } (L_{\lambda m})}$$

Hence along a meridian λ, radial scale factor,

$$RSF = \frac{L_{\lambda g}}{L_{\lambda m}} \quad \text{... (iii)}$$

and radial scale is expressed by

$$1 : RSF \quad \text{... (iv)}$$

Deformation

Along the two principal directions, it is the balance of the scale factors that determines the nature and magnitude of deformations on a projection. There are four principal types of deformations. These are deformations in *area*, *shape*, *distance* and *direction*, which are mutually exclusive in nature. On a projection transformation, scale factors are simple vectors, their products and resultants determine the specific property of a projection. On the basis of this, projections are classified into five types:

1. **Equal-Area Projections**

   In these, the area of a segment on the generating globe is truly preserved on the corresponding segment of the graticules. At any point of such projections, the product of the two scale factors is unity, or, in other words,

   $$RSF \times TSF = 1$$

   These are also called *authalic*, *homolographic* or *equivalent projections*.

2. **Orthomorphic Projections**

   Here, the shape of a segment on the generating globe is truly preserved on the corresponding segment of the graticules. At any point of such projections, the two scale factors are exactly equal in magnitude. The necessary condition of orthomorphism is, therefore, the equality of scales along the two principal directions, i.e.,

   $$RSF = TSF$$

   These are also known as *true-shape* or *conformal projections*.

3. **Equidistant Projections**

   In these, the distance between any two points on the generating globe is truly preserved between the corresponding points on the graticules.

4. **Azimuthal Projections**
Here the azimuth defining the directions between any two points on the generating globe is truly preserved between the corresponding two points on the graticules.

5. **Aphylactic Projections**
In these, neither of the above four properties is truly and fully preserved. Such projections are genetically neither azimuthal equidistant, nor equivalent and nor confor

Classification of Map Projection

Map projections are fundamentally classi based on the *extrinsic* and *intrinsic* properties. T extrinsic properties include the exoge parameters of transformation, i.e., the nature

Table 2.1 Classification of Map Projections

Criteria	Parameter	Classes/Sub-classes
1. EXTRINSIC	A. Datum Surface	I. Direct or Spheroidal Projection II. Double or Spherical Projection III. Triple Projection
	B. Plane or Surface of Projection	1st Order (plane): I. Planar; II. Conical; III. Cylindrical 2nd Order (aspect): a. Tangent; b. Secant; c. Polysuperficial 3rd Order (case): i. Normal; ii. Transverse; iii. Oblique
	C. Method of Projection	I. Perspective — a. Gnomonic; b. Stereographic; c. Orthographic II. Semiperspective III. Non-perspective IV. Conventional
2. INTRINSIC		I. Azimuthal II. Equidistant III. Equivalent or Authalic or Homolographic IV. Orthomorphic or Conformal V. Aphylactic
	B. Appearance of Parallels and Meridians	I. Both parallels and meridians are straight lines II. Parallels are straight lines and meridians are regular curves III. Parallels are regular curves and meridians are straight lines IV. Both parallels and meridians are regular curves V. Parallels are concentric circles and meridians are regular curves. VI. Parallels are concentric circles and meridians are radiating straight lines VII. Parallels are irregular curves and meridians are radiating straight lines VIII. Both parallels and meridians are irregular curves
	C. Geometric shape	I. Rectangular II. Circular III. Elliptical IV. Parabolic V. Butterfly VI. Others

the poles are 90°N and 90°S. It is measured either to the north or to the south of the equator and is accordingly specified as °N or °S. Through each latitude, circles parallel to the equator and centred on the polar axis may be imagined. These are called *parallels of latitudes* or simply *parallels*. Altogether there are 180 parallels at 1° intervals. Of the parallels only the equator is a great circle. The radius and the length of the parallels gradually decrease from its maxima at the equator to its minima at the poles.

The semicircular lines joining the two poles and intersecting the parallels at right angles are called *meridians* or *lines of longitudes*. All meridians are equal in size. There are 360 meridians at 1° intervals. Of these, the one that passes through Greenwich is taken as the reference line and is called the *prime meridian*. The longitude of a place is described as the angle subtended by the meridional plane passing through a place on the plane of the prime meridian, i.e., 0° at the centre of the earth. It is measured either to the east or to the west of the prime meridian and is accordingly specified as °E and °W.

Graticule

It refers to the net or mesh of mutually intersecting parallels and meridians drawn to a certain scale and based on certain principles. The term *graticulation* is used to specify the procedures by which the network of graticules are drawn.

Generating Globe

It refers to the globe from which projections are generated or developed. Normally it is a small skeleton globe made of glass or wire (Fig. 2.2). The parallels and meridians are shown by black lines (glass globe) or wires (wire globe) placed at their true angular distances apart. Naturally the generating globe is a geometrically accurate earth reduced in size.

Projection Plane

It is a 2-dimensional geometric plane upon which the parallels and meridians are usually projected. In case of a perspective planar projection, the

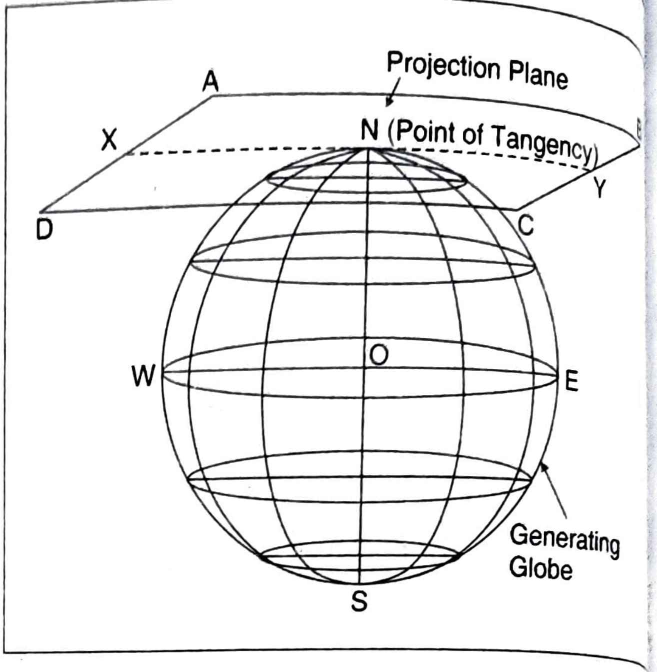

Fig. 2.2 Projection Plane and Generating Globe

projection plane touches the generating globe a single point (Fig. 2.2).

Developable Surface

In case of planar projections, only a single point truly represented with the exact one-to-o correspondence. Obviously, from this point tangency, the distortion on a map increases in directions. To minimise it, the point of conta with the generating globe is maximised by usin projection surfaces that can easily be develope into 2-dimensional geometric planes. Suc projection surfaces are known as *developabl surfaces*, e.g., a cone or a cylinder. A right circul cone or a cylinder usually touches a generatin globe along a parallel and may even intersect along two different parallels in certain desire situations (Fig. 2.3). Along these parallels, one-to-one correspondence is truly maintained involvin no error and are termed as *lines of zero distortio* When developed, a cone becomes a *sector o circle* and a cylinder becomes a *rectangl* both being parts of a 2-dimensional plane. Notably, when the angle at the vertex of a cone becomes 180°, the cone is developed into a projection plane touching the generating globe at a single point only. Again, when the apex of a cone lies at infinity, the cone is developed into a cylinder touching the generating globe along the equator.

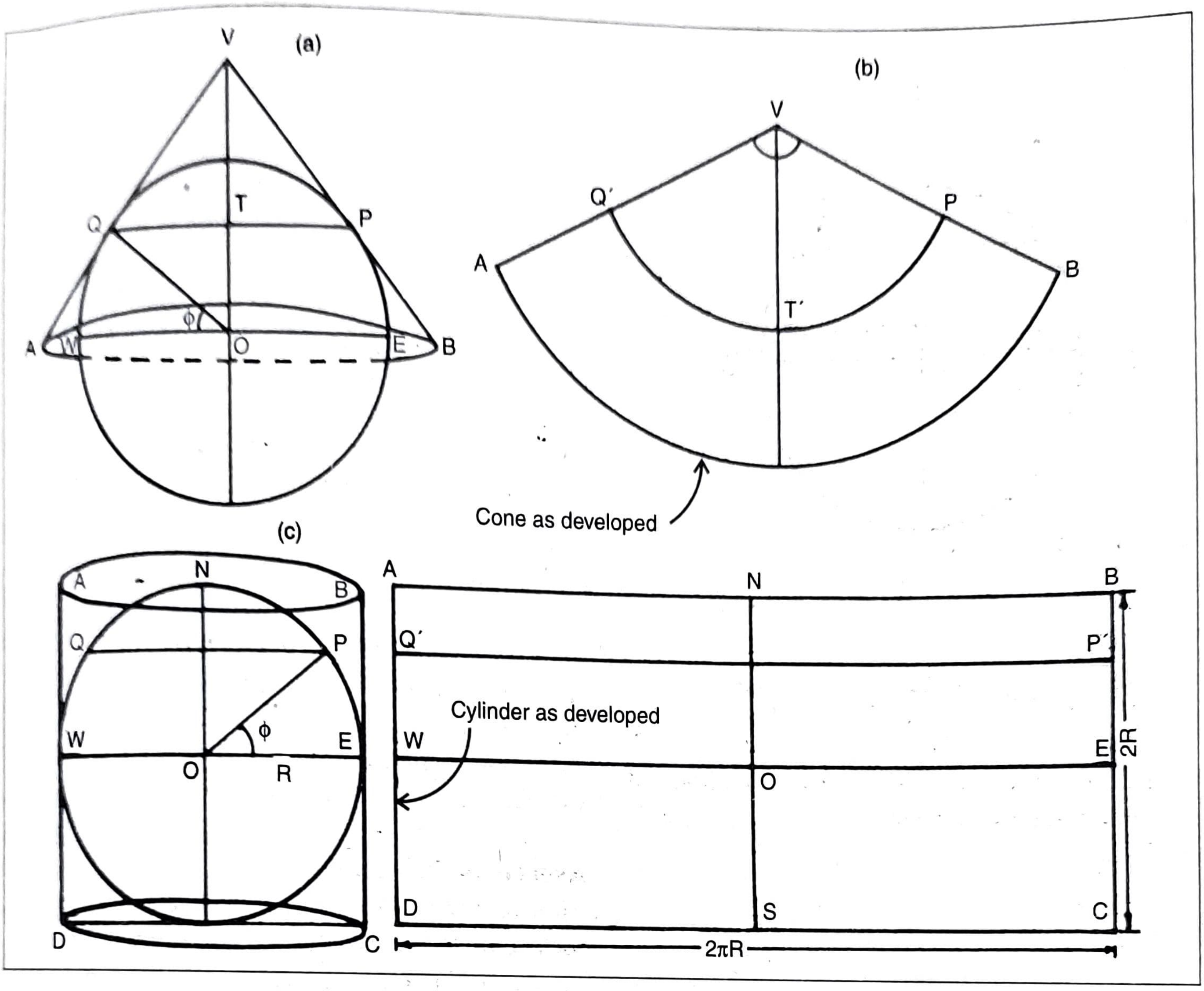

Fig. 2.3 Developable Surfaces: Cone and Cylinder

Central Meridian

For a given longitudinal extension, it refers to that meridian, *which lies exactly at the median* or *middle-most* position of that extension. It has only constructional importance and is normally drawn as a straight line. The mesh of graticules on one side of the central meridian (CM) is in fact the mirror image of the other side.

Standard Parallel

The parallel(s), *along which a projection plane or a developable surface touch*(es) or *intersect*(s) *the generating globe*, are called *standard parallel*(s). Along the standard parallels, the tangential scale is essentially 1:1. Hence, these are always the *lines of zero distortion.*

Constant of a Cone

It is defined as the *ratio between the angle at the vertex* or apex of a cone when developed (α) and the angle at the pole of the generating globe (360°) (Fig. 2.4).

Therefore, the constant of a *cone*, $n = \frac{\alpha}{360°}$

Since α depends on the standard parallel (ϕ_0), n is a direct function of ϕ_0. The two extreme situations are:

i. when $\phi_0 = 0°$, a cone is transformed into a special cylinder with $\alpha = 0°$.

Therefore, $n = \frac{0°}{360°} = 0$ ... (i)

ii. when $\phi_0 = 90°$, a cone is transformed into a plane and α becomes 360°.

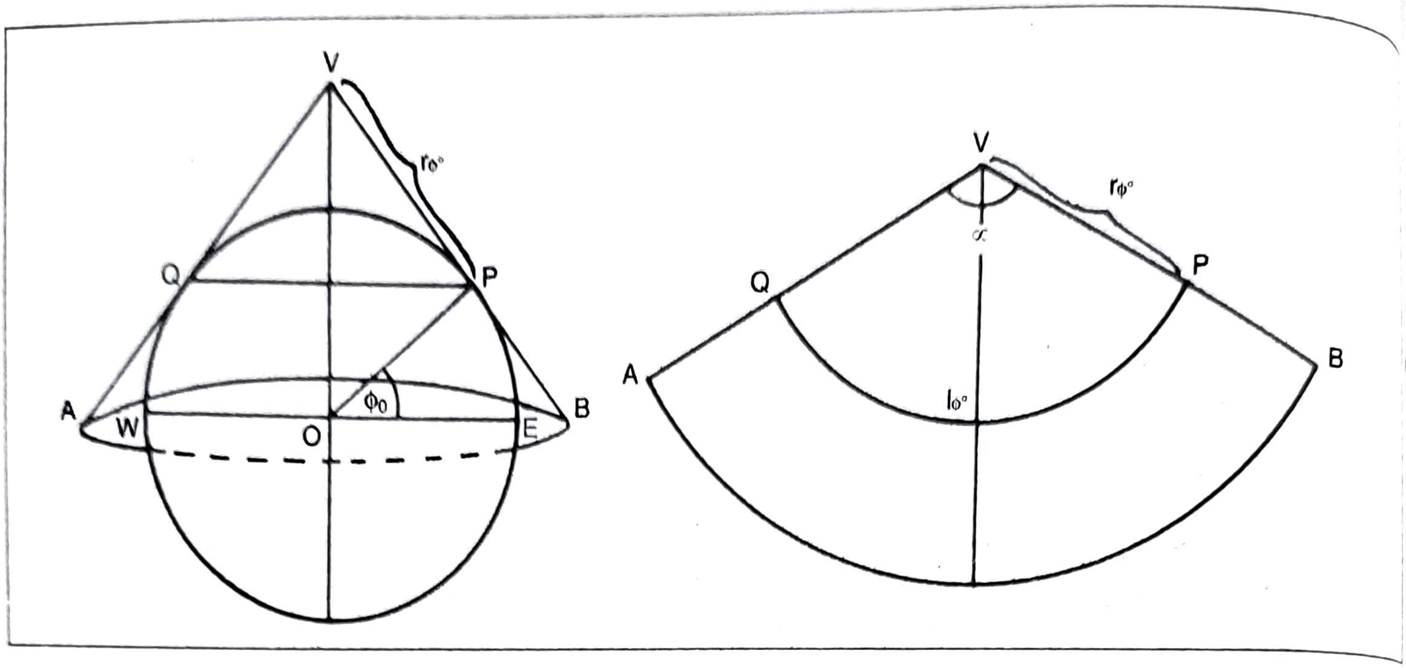

Fig. 2.4 Constant of a Cone

$$\text{Therefore, } n = \frac{360^\circ}{360^\circ}$$

$$= 1 \qquad \text{... (ii)}$$

Hence, $0 \le n \le 1$ is the boundary condition of the constant of a cone.

From Fig. 2.4,

$$\alpha . r_{\phi_0} = l_{\phi_0}$$

$$\text{or, } \alpha = \frac{l_{\phi_0}}{r_{\phi_0}}$$

Therefore, n

$$= \frac{\alpha}{360^\circ}$$

$$= \frac{l_{\phi_0}}{360^\circ . r_{\phi_0}}$$

$$= \frac{2\pi . R . \cos\phi_0}{360^\circ . R . \cot\phi_0} \left[\begin{array}{l} \therefore l_{\phi_0} = 2\pi . R . \cos\phi_0 \\ r_{\phi_0} = R . \cot\phi_0 \end{array} \right]$$

$$= \sin\phi_0 .$$

Thus, in the simplest case of a conical projection with one standard parallel, the constant of a cone is equal to the *sine* value of the standard parallel.

Cases of Projection

This is defined as the geometric relation expressed as the angle between the axis of symmetry of the plane of projection and the polar axis of the globe, (β). There can be three cases of projection—*normal, transverse* and *oblique*. In the normal case, $\beta = 0^\circ$, in the transverse case $\beta = 90^\circ$ and in the oblique case $90^\circ > \beta > 0^\circ$.

Aspects of Projection

This refers to the *attitude* of the plane or the surface of projection. The plane of projection may be tangent at one point only or intersect along a parallel circle. A polyhedric surface, (i.e., one which has more than one plane) may be tangent at a number of points. Similarly, a cone or a cylinder may be tangent along a parallel or may interest along two parallels. A polyconic or polycylindrical surface may even be chosen for a projection. The main objective is to maximise the points of contact in order to minimise the cumulative deformation.

Perspective Projections

In these, graticules are drawn from a transparent generating globe made of glass with the help of a light source. Rays emerging from the sources cast shadows of parallels and meridians on the projection plane, e.g., *Gnomonic projection, Stereographic projection, Orthographic projection* and the *Simple Conic projection with 1 standard parallel.*

Semi-perspective Projections

In these, one set of intersecting lines is geometrically projected and the other set drawn purely to suit a desired property.

Non-perspective Projections

In these, projection is done in accordance to a consistent mathematical principle to satisfy certain objectives.

Conventional Projections

These are non-perspective projections constructed following a set of conventions purely based on mathematical operations postulated by a cartographer to portray the whole globe with certain objectives.

The Great Circle

If a plane intersects a sphere, the resulting section of the curved surface which is traced on the plane, is a circle. If the intersecting plane passes through the center of a sphere, the resulting section is a circle, whose radius is the largest which can occur and is equal to the radius of the sphere itself. This is defined as a *great circle*. Thus a meridian is a part of a great circle. The equator is the only parallel which is a great circle and all other parallels are *small circles*. If the plane does not pass through the centre of the sphere, the radius of the resulting circle is less than that of the sphere. This is called a small circle. The special features of great circle are:

- The axis of two or more great circles cannot coincide.
- Intersecting great circles bisect each other.
- The plane of a great circle divides a sphere into two equal halves.
- The section of all great circles passes through the centre of the sphere; therefore the centre of the sphere is the common centre of all the great circles.
- Only one great circle can be drawn through any two points on the spherical surface which are not diametrically opposite to one another.
- An infinite number of great circles can be drawn through a single point.
- An infinite number of great circles can be drawn on a sphere.
- The shorter arc of the great circle through two points is the shortest distance between the points on the spherical surface.

The Geodesic

Similar to the great circle arc, the shortest possible connection between two points on the ellipsoidal surface is defined as the *geodetic line* or, for convenience, the *geodesic*. Progressing along this curved line from point to point, the tangent continuously changes its azimuth. According to Clairaut's theorem, '*the product of the radius* (r) *of the parallel circle* (ϕ) *and the sine of the azimuth* (α) *of the geodesic is a constant*'. Therefore,

$$r \sin\alpha = (R \cos\phi).\sin\alpha$$

$$= k \text{ (a constant)} \quad \ldots (1)$$

The following particulars can be derived from this,

1. For $\phi = 0°$, $R = a$ and $\sin\alpha = k/a$. Hence, the geodesic intersects the equator with azimuth $\alpha = \sin^{-1}(k/a)$

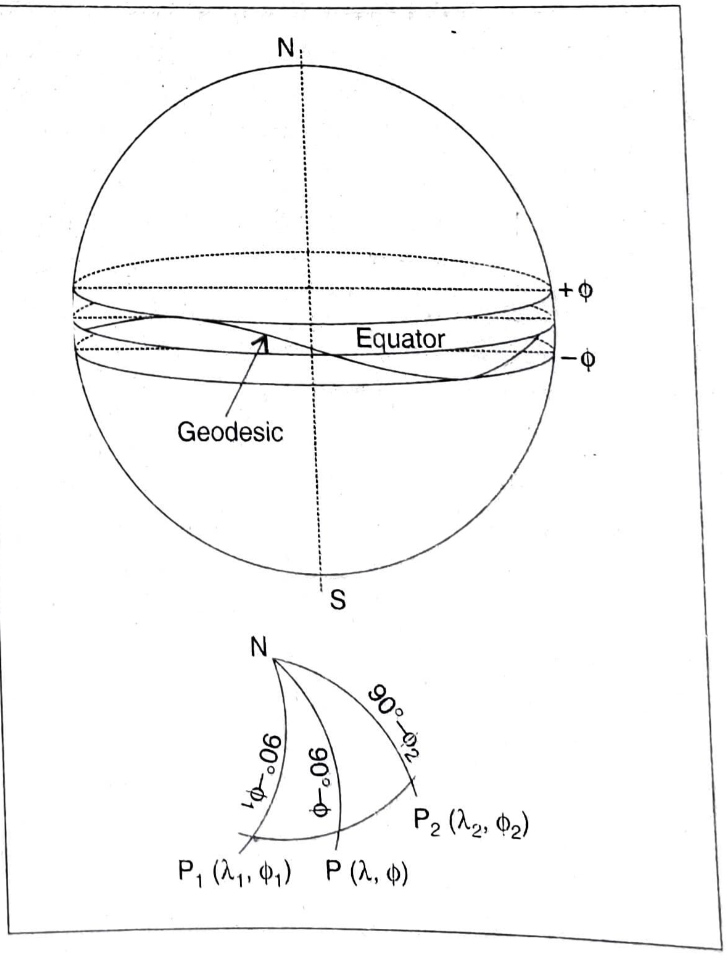

Fig. 2.5 The Geodesic

Theory

i. Radius of the generating globe, R = Actual radius of the earth ÷ Denominator of R.F.

ii. Radius of any parallel (ϕ), r_ϕ = R.cot ϕ

Example

Draw graticules at 10° interval on scale 1:234 × 10^6 for the extension 90°S-40°S around the pole.

Calculation

i. $R = \dfrac{640\times10^6 \text{ cm}}{234\times10^6}$

$= 2.73$ cm

ii. $r_\phi = 2.73 \cot\phi$ (Table 2.2)

Construction

i. A pair of straight lines intersecting at right angles are drawn to represent the four cardinal meridians (0°, 90°E, 180°, 90°W).

ii. From the point of intersection, concentric circles are drawn with r_ϕ to represent the parallels.

iii. With the help of a protractor held at the pole, division points are marked at the required interval of angle.

iv. Straight lines are drawn through these points joining the poles to represent the meridians.

v. The graticules are then properly labelled (Fig. 2.8).

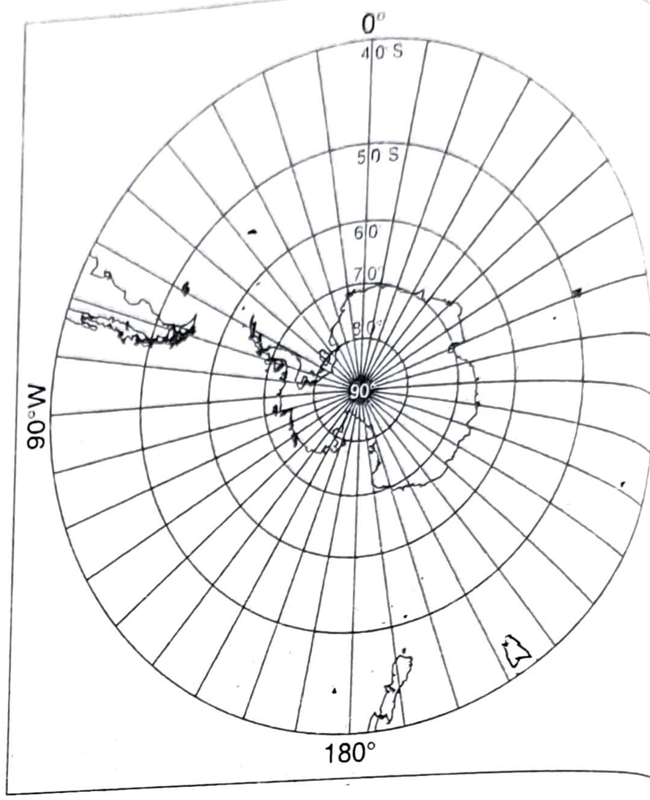

Fig. 2.8 Polar Zenithal Gnomonic Projection

Properties

i. Parallells are concentric circles.

ii. Interparallel spacing increases rapidly towards the equator.

iii. The equator cannot be represented in this projection.

iv. Meridians are straight lines radiating from the poles at true angular distances apart.

v. It is an azimuthal projection as the azimuth of a point at the poles is truly maintained.

vi. All great circles appear as straight lines as their planes pass through the centre of the globe, where the source of light is kept.

vii. The shortest distance between two points is represented by a straight line while a rhumbline is represented by a *nautilus* (i.e., a curve analogous to that of a snail's back).

viii. Deformation increases rapidly towards the margin of the map.

ix. It is useful to the navigators and is suited to small areas around the pole.

Polar Zenithal Stereographic Projection

Principle

In this projection, a 2-dimensional plane of projection touches the generating globe at either of the poles. It is a perspective projection, with the source of light lying at the pole diametrically opposite to one at which the projection plane touches the generating globe (Fig. 2.9). The parallels are projected as concentric circles of

Table 2.2 Computation of r_ϕ

ϕ	40° S	50° S	60° S	70° S	80° S	90° S
$r_\phi = 2.73 \cot\phi$ (cm)	3.25	2.29	1.57	0.99	0.48	0

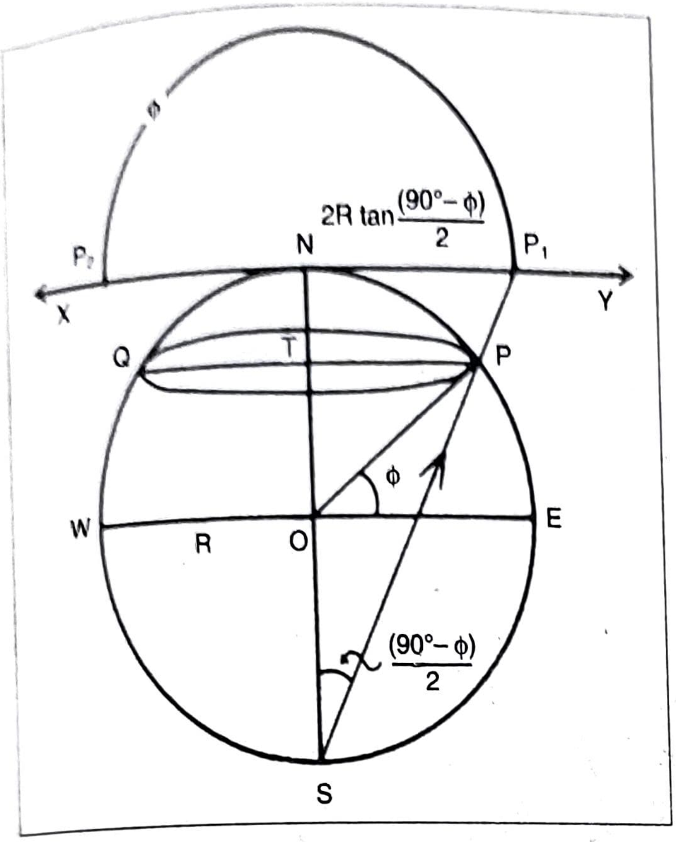

Fig. 2.9 Principles of Polar Zenithal Stereographic Projection

varying radius while the meridians are projected as straight lines radiating from the poles.

In Fig. 2.9, let XY be the projection plane and S is the source of light. Parallel PQ is projected on XY as a circle of radius NP_1 or NP_2

Radius CE = CP = CN = CW = CS = R

In ΔCPS, CP = CS = R

∴ it is an isosceles triangle

and $\angle CSP = \angle CPS$

$\angle NCP = \angle NCE - \angle PCE$

$= (90° - \phi)$

∠NCP being an external angle at C of ΔCPS,

$\angle CPS + \angle CSP = \angle NCP$

or, $2\angle CSP = (90° - \phi)$

or, $\angle CPS = \left(\frac{90° - \phi}{2}\right)$

XY is tangent at N.

∴ ΔNP_1S is a right angled triangle

and $\frac{NP_1}{NS} = \tan \angle CSP$

or, $NP_1 = NS.\tan \angle CSP$

$= 2R.\tan\left(\frac{90° - \phi}{2}\right)$

∴ Radius of any parallel $\phi = 2R.\tan\left(\frac{90° - \phi}{2}\right)$

Theory

i. Radius of the generating globe. R = Actual radius of the earth ÷ Denominator of R.F.

ii. Radius of any parallel (φ).

$$r_\phi = 2R.\tan\left(\frac{90° - \phi}{2}\right)$$

Example

Draw graticules at 10° interval on scale 1:184 × 10^6 for the extension 90°S-40°S around the pole.

Calculation

i. $R = \frac{640 \times 10^6 \text{ cm}}{184 \times 10^6}$

$= 3.84$ cm

ii. $r_\phi = 2 \times 3.84 \tan\left(\frac{90° - \phi}{2}\right)$

$= 6.96 \tan\left(\frac{90° - \phi}{2}\right)$ (Table 2.3)

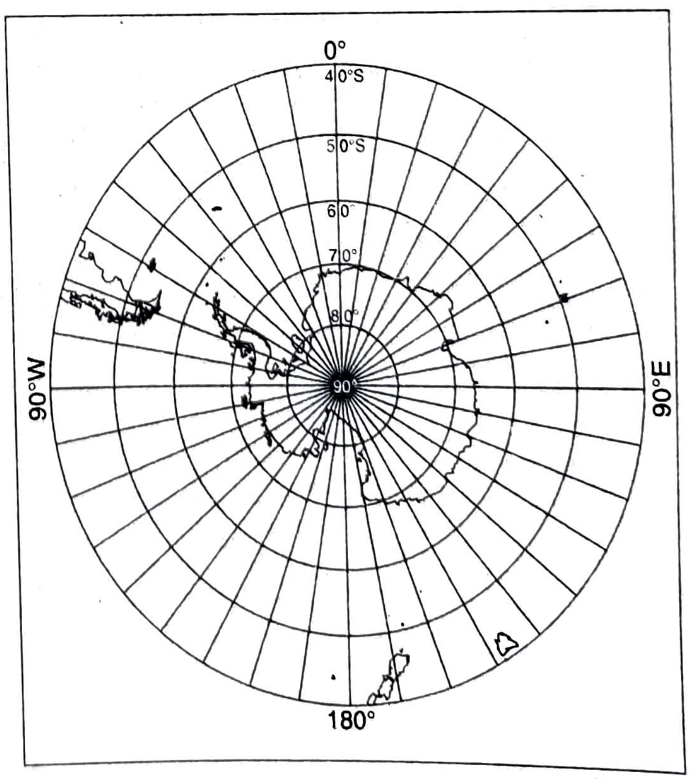

Fig. 2.10 Polar Zenithal Stereographic Projection

Table 2.3 Computation of r_ϕ

ϕ	40° S	50° S	60° S	70° S	80° S	[illegible]
$r_\phi = 6.96 \tan\left(\frac{90° - \phi}{2}\right)$ cm	3.25	2.53	1.86	1.23	0.61	[illegible]

Construction

Similar to Polar Zenithal Gnomonic Projection (Fig. 2.10).

Properties

i. Parallels are represented by concentric circles of varying radii.
ii. Inter-parallel spacing gradually increases toward the equator.
iii. Meridians are straight lines radiating from the poles at true azimuth apart.
iv. The direction between the two points is maintained.
v. At any point, the radial scale is equal to the tangential scale.
vi. It is an orthomorphic projection, i.e., the shape of a map is truly maintained.
vii. It is commonly used for the map of the *world* in hemispheres.

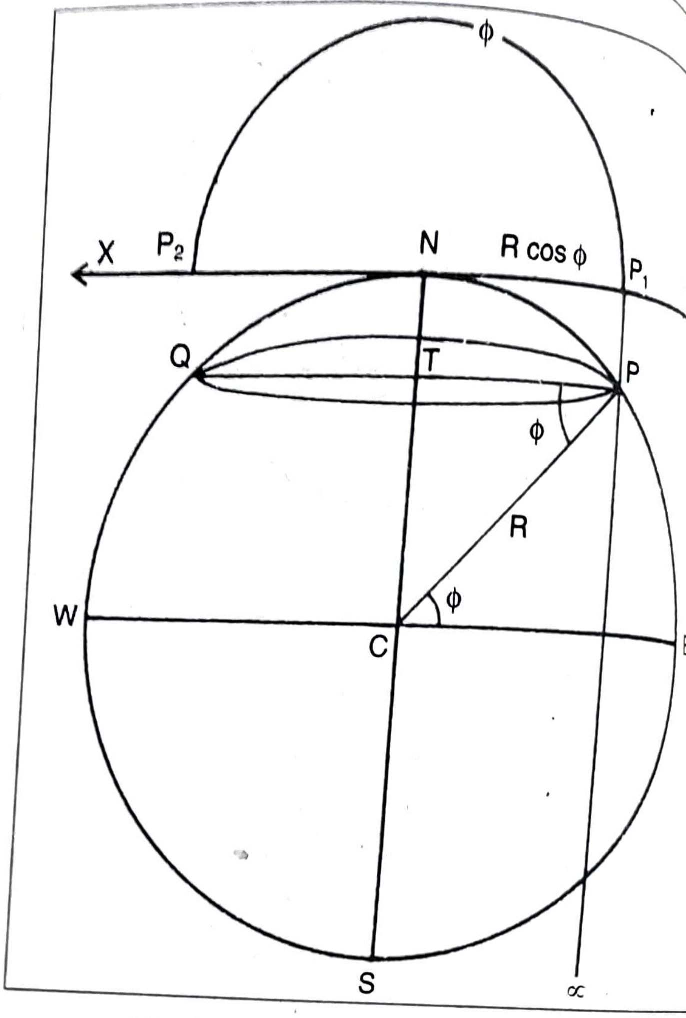

Fig. 2.11 Principles of Polar Zenithal Orthographic Projection

Polar Zenithal Orthographic Projection

Principle

In this projection, a 2-dimensional plane of projection touches the generating globe at either of the poles. It is a perspective projection and the source of light lies at infinity (Fig. 2.11). Rays of light passing through the parallels become incident on the projection plane at right angles. The parallels are projected as concentric circles of varying radius while the meridians are projected as straight lines at true azimuth apart at the poles.

In Fig. 2.11, let XY be the projection plane. The source of light is at infinity (µ). Parallel PQ is projected on XY as a circle of radius NP_1.

Radius, CE = CW = CP = CN = CS = R

XY is tangent at N

$\because$ XY || PQ || WE,

$NP_1 = PT$ = radius of the projected parallel ϕ an[d]

$\angle TPC = \angle PCE = \phi$

From rt $\angle\Delta TPC$, $\dfrac{PT}{PC} = \cos\phi$

or, $PT = PC.\cos\phi$

$= R\cos\phi$

$\therefore$ radius of any parallel $\phi = R\cos\phi$

Theory

i. Radius of the generating globe, R = Actual radius of the earth ÷ Denominator of R.F.
ii. Radius of any parallel (ϕ), $r_\phi = R\cos\phi$

Construction

Similar to Polar Zenithal Gnomonic Projection (Fig. 2.12).

Table 2.6 Computation of r_ϕ

ϕ	30° S	40° S	50° S	60° S	70° S	80° S	90° S
$r_\phi = 7.66 \sin\left(\frac{90^\circ - \phi}{2}\right)$ cm	3.83	3.24	2.62	1.98	1.33	0.67	0

iii. Interparallel spacing decreases gradually toward the equator.
iv. Meridians are straight lines radiating from the poles at true angular distances apart.
v. It is an azimuthal projection as the azimuth of any point at the poles is truly maintained.
vi. It is an equal-area projection as at any point the product of tangential and radial scale is unity.
vii. It is most commonly used to show polar areas in a world atlas. (Fig. 2.16).

Example

Draw graticules at 10° interval on scale 1:167 × 10^6 for the extension 40°S-90°S around the pole.

Calculation

i. $R = \frac{640 \times 10^6 \text{ cm}}{167 \times 10^6}$

$= 3.83$ cm.

ii. $r_\phi = 2 \times 3.83 . \sin\left(\frac{90^\circ - \phi}{2}\right)$

$= 7.66 \sin\left(\frac{90^\circ - \phi}{2}\right)$

Simple Conical Projection with I Standard Parallel

Principle

In this projection, a simple right circular cone touches the generating globe along a parallel. This is the parallel along which distortions of any kind is nil and is known as the *standard parallel*. It is a perspective projection in which the parallels and meridians are projected directly on the inner surface of the cone with respect to a light source at the centre of the generating globe (Fig. 2.17).

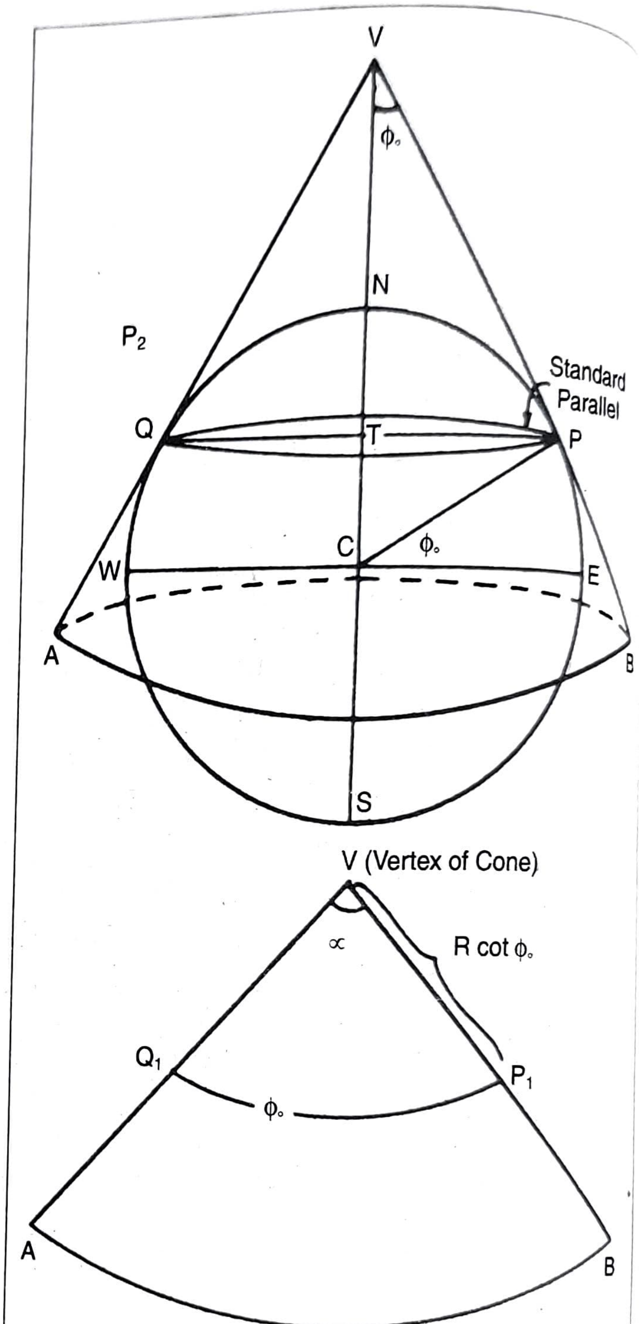

Fig. 2.17 Principles of Simple Conical Projection with I Standard Parallel

In Fig. 2.17, let cone VAB be the projection plane which touches the generating globe along PQ (ϕ_0). Therefore standard parallel PQ is projected as an arc of circle with radius VP or VP_1.

Radius, CE = CP = CN = CQ = CW = CS = R

VB is tangent at P and

$\therefore \angle VPC = \angle CPB = 90°$

and ΔVPC is a right angled triangle

$\angle PVC = [180° - (\angle VPC + \angle VCP)]$

$= [180° - (90° + \angle VCP)]$

$= (90° - \angle VCP)$

$= \angle VCE - \angle VCP\ [\because \angle VCE = 90°]$

$= \angle PCE$

$= \phi_0$

From the rt $\angle \Delta PVC$, $\dfrac{VP}{PC} = \cot\phi_0$

or, $VP = PC.\cot\phi_0$

$= R\cot\phi_0$

$\therefore$ Radius of the standard parallel, $r_0 = R\cot\phi_0$

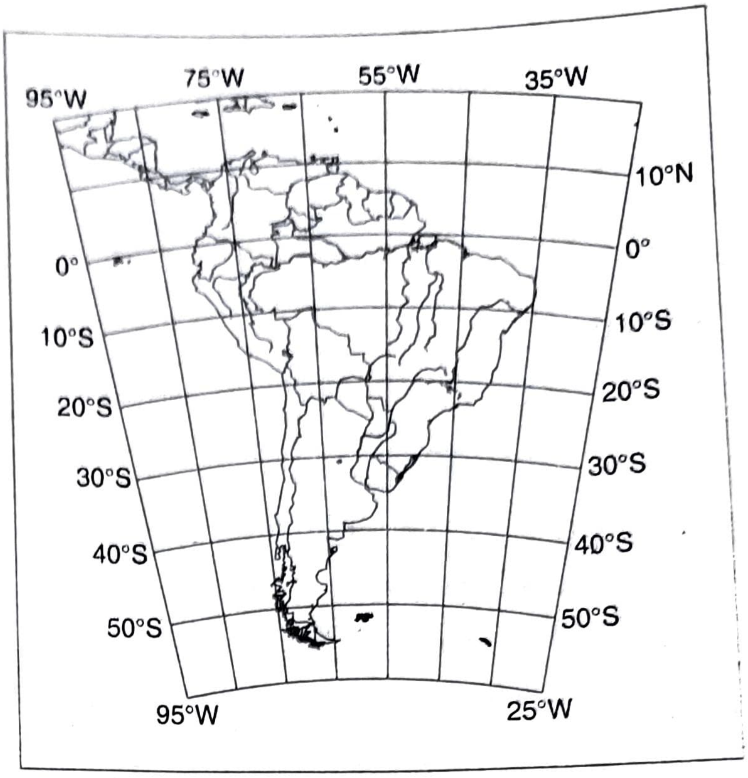

Fig. 2.18 Simple Conical Projection with I Standard Parallel

Theory

i. The radius of the generating globe, R = Actual radius of the earth ÷ Denominator of the R.F.

ii. The division of the central meridian for spacing the parallels at i° interval,

$$d_1 = \frac{\pi R}{180°} \times i°$$

iii. The radius of the standard parallel (ϕ_0), $r_0 = R\cot\phi_0$

iv. The division on the standard parallel for spacing the meridians at i° interval,

$$d_2 = \frac{2\pi R\cos\phi_0}{360°} \times i°$$

Construction

i. A straight line is drawn vertically through the centre of the paper to represent the central meridian.

ii. It is then divided by d_1 for spacing the parallels.

iii. An arc of circle is then drawn through the standard parallel mark with radius and centre on the central meridian (produced if necessary).

iv. Concentric arcs of circle are then drawn through each division on the central meridian to represent other parallels.

v. The standard parallel is divided by d_2 on both sides on the central meridian for spacing the meridians.

vi. Straight lines are drawn through each of these division points joining the centre of the arcs to represent the meridians.

vii. The graticules are then properly labelled (Fig. 2.18).

Properties

i. In the generic sense, this is a perspective projection. The parallels are concentric arcs of circles truly spaced on the central meridian.

ii. Poles are also represented by arcs in this projection.

iii. The radial scale is true along all the meridians.

iv. Meridians are straight lines truly spaced on the standard parallel and converging at the vertex of the cone.

v. The tangential scale is true along the standard parallel only.

vi. Positive deformation occurs on the equator-ward segment while negative deformation occurs on the pole-ward segment away from the standard parallel.

vii. It is an aphylactic projection, i.e., one that maintains neither area nor shape.

viii. It is suitable for smaller countries of mid-latitude or temperate regions.

Example

Draw graticules at 10° interval on scale 1:149 × 10^6 for the extensions, 20°N-60°S and 25°W-95°W.

Calculation

i. $R = \dfrac{640 \times 10^6 \text{ cm}}{149 \times 10^6}$

$= 4.30$ cm

ii. Parallels to be drawn: 20°N, 10°N, 0°, 10°S, 20°S, 30°S, 40°S, 50°S, 60°S. The standard parallel (ϕ_0) chosen is 20°S.

iii. $d = \dfrac{\pi . 4.30}{180} \times 10°$

$= 0.75$ cm

iv. $r_0 = 4.30 \cot 20°$

$= 11.81$ cm

v. $d_1 = \dfrac{2\pi . 4.30 \cos 20°}{360°} \times 10°$

$= 0.71$ cm

Simple Conical Projection with II Standard Parallels

Principle

In this projection, a simple right circular cone is taken as the projection plane. Two circles of the cone correspond to two different parallels on the generating globe and form an ordinary cone independent of the globe (Fig. 2.19). These are the standard parallels which are so selected as to cover two-thirds of the latitudinal extent of the area to be mapped. The parallels appear as concentric arcs of a circle while the meridians appear as straight lines converging at the vertex of the cone.

In Fig. 2.19, standard parallels MN(ϕ_1) and PQ(ϕ_2) are projected as arcs of circle M_1N_1 and P_1Q_1 with radii r_1 and r_2 respectively

$VP_1.\alpha$ = arc P_1Q_1

= true length of ϕ_2 parallel on globe

or, $r_2\alpha = 2\pi R \cos\phi_2$

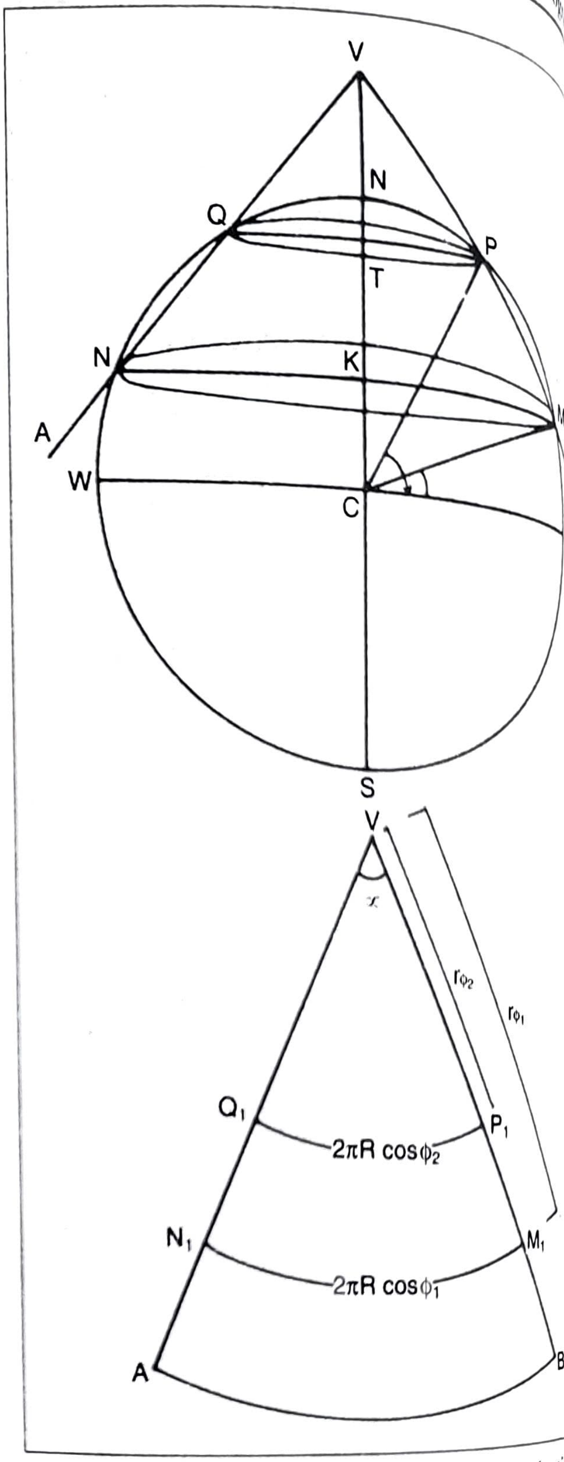

Fig. 2.19 Principles of Simple Conical Projection with II Standard Parallels

$\therefore r_2 = \dfrac{2\pi R \cos\phi_2}{\alpha}$

$VM_1.\alpha$ = arc M_1N_1

= true length of ϕ_1 parallel on globe

or, $r_1.\alpha = 2\pi R \cos\phi_1$

or, $r_1 = \dfrac{2\pi R \cos\phi_1}{\alpha}$

Now,

$r_1 - r_2 = VM_1 - VP_1$

$= M_1P_1$

Table 2.13 Computation of r_ϕ

ϕ	20° N	30° N	40°N	50° N	60° N	70° N	80°
$r_\phi = 5.77 \cot \phi$ (cm)	15.85	9.99	6.88	4.84	3.33	2.10	[illegible]

Table 2.14 Computation of d_ϕ

ϕ	20° N	30° N	40°N	50° N	60° N	70° N	80°
$d_\phi = 1.00705 \cos \phi$ (cm)	0.95	0.87	0.77	0.65	0.50	0.34	[illegible]

ii. $d = \frac{\pi \times 5.77}{180} \times 10 \Rightarrow 1.097$ cm

iii. $r_\phi = 5.77 \cot\phi$ cm (Table 2.13)

iv. $d_\phi = \frac{2\pi \times 5.77 \cos\phi}{360°} \times 10°$

$= 1.00705 \cos\phi$ cm (Table 2.14)

Cylindrical Equal-Area Projection

Principle

Lambert developed this projection in which a simple right circular cylinder touches the globe along the equator. Parallels and meridians are both projected as straight lines intersecting one another at right angles (Fig. 2.35). Tangential scale along all the parallels is kept equal to that along the equator. To maintain true area, radial scale along a meridian is made reciprocal to the tangential scale at that point. Hence, parallels lie at different heights above the equator. The interparallel spacing decreases rapidly towards the poles as parallels are all of same length as the equator.

In Fig. 2.33, let the cylinder ABCD touch the globe along the equator.

The parallel PQ is projected as straight line at PM distance away from WE.

$P_1Q_1 \parallel WE$ and $\angle POM = \phi$

Length of parallel (ϕ) on globe $= 2\pi R \cos\phi$.

Length of parallel (ϕ) on projection $= 2\pi R$

$$\therefore \text{tangential scale} = \frac{2\pi R}{2\pi R \cos\phi}$$

$$= \sec\phi \qquad \text{... (i)}$$

Let S be another parallel at $d\phi$ angle away from $P(\phi)$

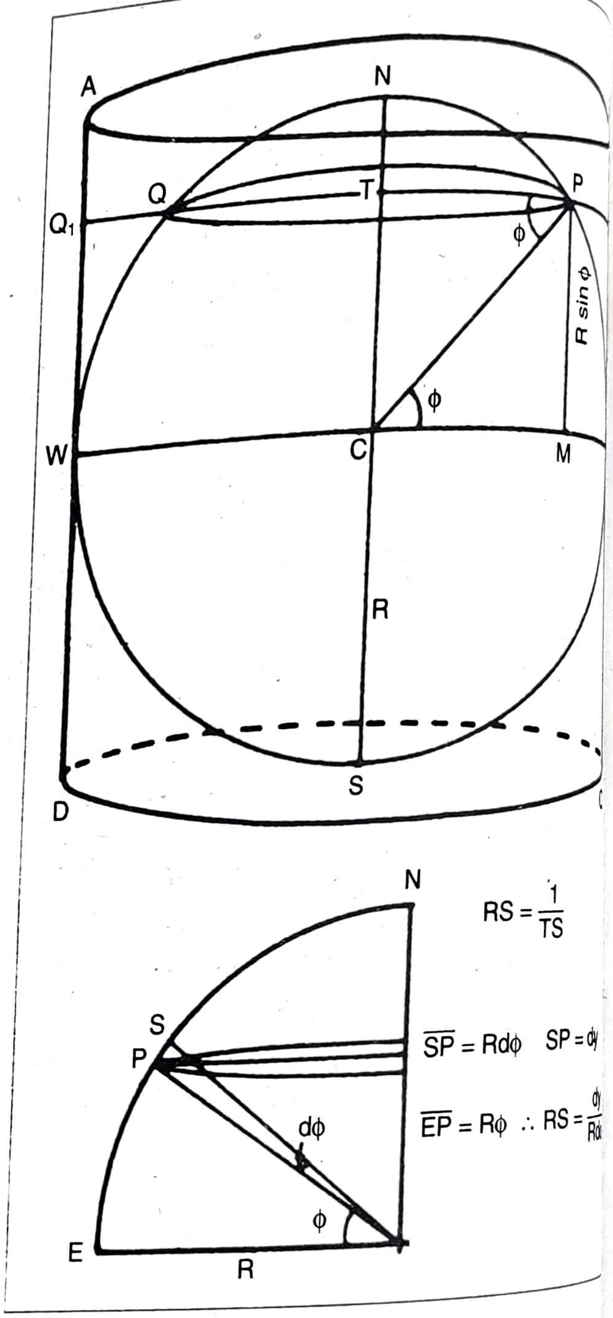

Fig. 2.35 Principles of Cylindrical Equal-Area Projection

∴ the true angular distance of $d\phi$ on globe = R.d
Let dy be the corresponding linear distance of d
from the equator on projection.

$\therefore$ radial scale $= \frac{dy}{R.d\phi}$ ... (ii)

Since it is an equal area projection,
radial scale × tangential scale = 1

or. $\frac{dy}{R.d\phi}.\sec\phi = 1$

or. $dy = R\cos\phi.d\phi$

By integration.

$\int dy = R\int \cos\phi.d\phi$

$\therefore \quad y = R\sin\phi$

Theory

i. Radius of the generating globe, R = Actual radius of the earth ÷ Denominator of R.F.
ii. Division along the equator for spacing the meridians at i° interval,

$$d = \frac{2\pi R}{360°} \times i°$$

iii. Height of any parallel above equator,

$$y_\phi = R\sin\phi$$

Construction

i. A straight line is drawn horizontally through the centre of the paper to represent the equator.
ii. It is then divided by d for spacing the meridians.
iii. Through each of these division points, straight lines are drawn perpendicular to the equator to represent the meridians.
iv. On the central meridian, heights of different parallels (y_ϕ) from the equator are marked.
v. Through each of these points, straight lines are drawn perpendicular to the central meridian to represent the parallels.
vi. The graticules are then properly labelled (Fig. 2.36).

Properties

i. Parallels are represented by a set of parallel straight lines.
ii. Parallels are of same the length as the equator (2πR).
iii. Parallels are variably spaced on the meridians.
iv. Interparallel spacing decreases rapidly toward the pole.
v. The tangential scale rapidly increases poleward and is infinity at the poles.
vi. Meridians are parallel straight lines truly spaced on the equator.
vii. Meridians are of same length equal to the diameter of the globe (2R).
viii. The intermeridian spacing is uniform on all the parallels.
ix. The pole is represented by a straight line of length 2πR.

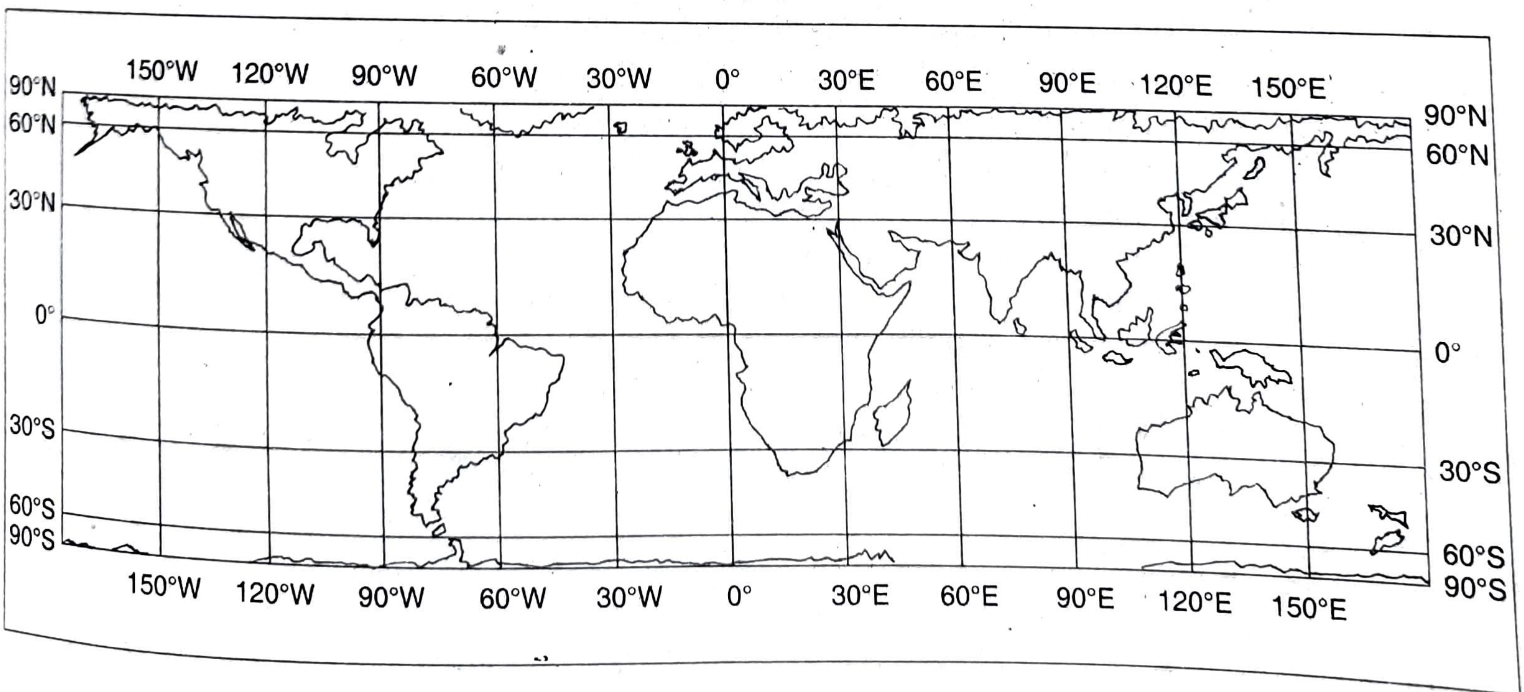

Fig. 2.36 Cylindrical Equal-Area Projection

Table 2.15 Computation of y_ϕ

ϕ	30° N/S	60° N/S	90° N/S
$y_\phi = 2.15 \sin\phi$ (cm)	1.075	1.862	2.150

x. At any point, the product of the two principal scales is unity.
xi. It is an equal-area projection.
xii. The shape is largely distorted near the poles.

Example
Draw graticules at 30° interval on scale, 1:297 × 10^6 for the whole globe.
Calculation

i. $R = \dfrac{640\times10^6 \text{ cm}}{297\times10^6} \Rightarrow 2.15$ cm

ii. $d = \dfrac{2\pi\times2.15}{360°}\times30° \Rightarrow 1.128$ cm

iii. $y_\phi = 2.15 \sin\phi$ cm (Table 2.15).

Mercator's Projection

Principle
This is a cylindrical orthomorphic projection designed by Flemish, Mercator and Wright. In this, a simple right circular cylinder touches the globe along the equator. All the parallels are of the same length equal to that of the equator and the meridians are equispaced on the parallels (Fig. 2.37). Therefore, the tangential scale increases infinitely toward the pole. To maintain the property of orthomorphism, the radial scale is made equal to the tangential scale at any point. Hence, parallels are variably spaced on the meridians and the poles can never be represented. The parallels and meridians are represented by sets of straight lines intersecting at right angles. In Fig. 2.37, let the cylinder ABCD touch the globe along the equator. The parallel, PQ, is projected as straight line at PM distance away from WE. PQ || WE and ∠POM

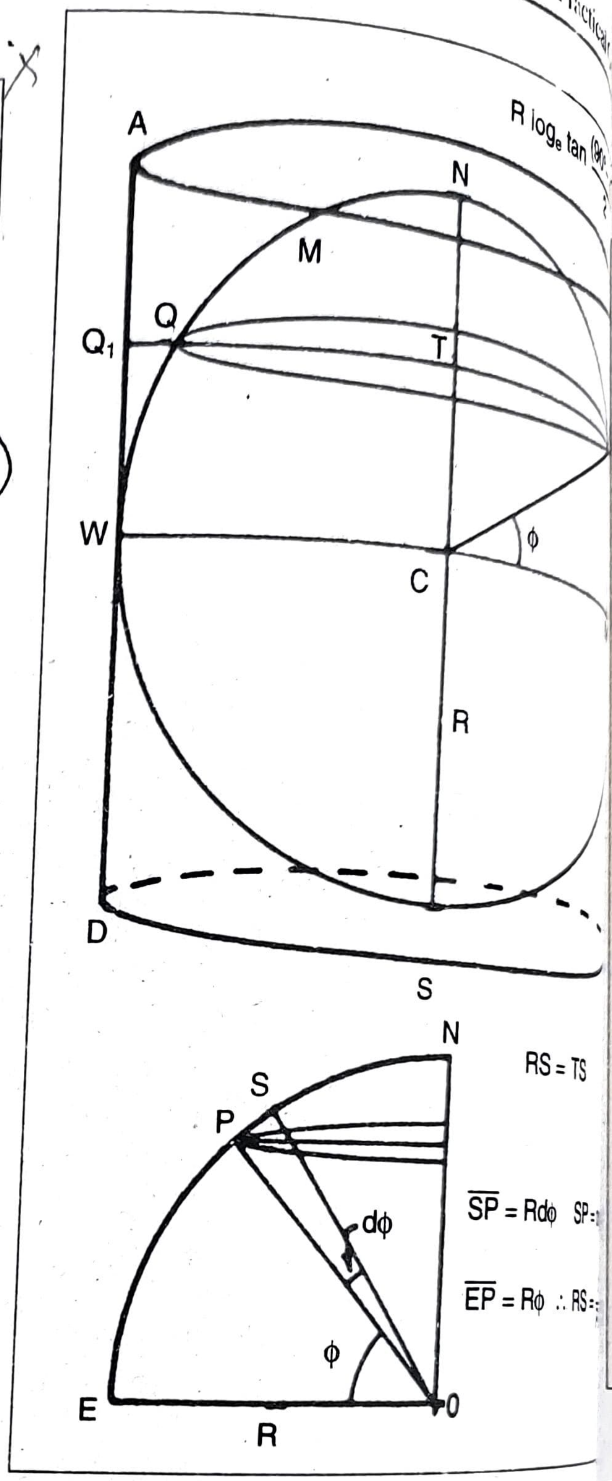

Fig. 2.37 Principles of Mercator's Projection

Let dy be the corresponding linear distance from the equator on projection.

$\therefore$ radial scale $= \dfrac{dy}{R.d\phi}$

Length of parallel (ϕ) on globe = $2\pi R \cos\phi$
Length of parallel (ϕ) on projection = $2\pi R$

$\therefore$ tangential scale $= \dfrac{2\pi R}{2\pi R \cos\phi}$

$= \sec\phi$