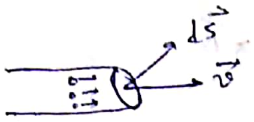
MAGNETOSTATIC FIELD

* Expression for current density & drift velocity ($\vec{J} = Ne\vec{v}$)



Let the drift velocity of the charge carriers be $\vec{v}$ in the direction of the applied electric field.

We consider a cylinder of area ds

So, volume covered by the charge carriers through the area ds in 1 sec $= \vec{v}.\vec{ds}$

If N be the no. density i.e., the number of charge carriers per unit volume, then

the number passing through the area $ds = N\vec{v}.\vec{ds}$

If e is the charge per carrier, then charge passing through area $\vec{ds}$ per sec

$$= dq = Ne\vec{v}.\vec{ds}$$

But charge dq passing per sec through ds is the current $dI = \vec{J}.\vec{ds}$ [J is current density]

$$\therefore \vec{J}.\vec{ds} = Ne\vec{v}.\vec{ds}$$

$$\therefore \boxed{\vec{J} = Ne\vec{v}}$$

Force on a linear current element

We consider an element dl of a current carrying conductor and calculate the force acting on it.

We have Lorentz force on charge q is $\vec{F}_m = q\vec{v} \times \vec{B}$

If N be the charge per unit volume, we have force acting on this volume $\vec{F}_m = Nq\vec{v} \times \vec{B}$.

So, total force acting on the charges crossing a finite volume V enclosed by the area ds is

$$\vec{F} = \int (Ne\vec{v} \times \vec{B})\, dV$$

$$= \int (Ne\vec{v} \times \vec{B})(\vec{dl}.\vec{ds})$$

$$= \int (\vec{J} \times \vec{B})(\vec{dl} \cdot \vec{ds})$$

$$= \int (\vec{J} \cdot \vec{ds})(\vec{dl} \times \vec{B}) = \int I \vec{dl} \times \vec{B}$$

$$\therefore \boxed{\vec{F} = \int I \vec{dl} \times \vec{B}}$$

When a closed loop is considered $\oint \vec{dl} = 0$, So, the above eqn. shows that force on a closed current loop is zero when B is uniform.

Biot-Savart law [Magnetic field of a steady current]

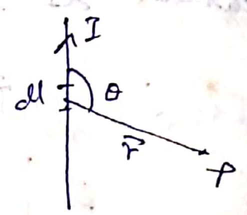
It is the relation for the magnetic field by the current element and is based upon the experimental facts.

The law states that the magnetic field B at a point P due to a very small element of length dl on a wire carrying current dI is

(i) directly proportional to the current I

(ii) directly proportional to dl & to $\sin\theta$, where θ is the angle between the vector $\vec{dl}$ and $\vec{r}$, the distance of P from dl

(iii) inversely proportional to r^2



$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I \vec{dl} \times \vec{r}}{r^3}$$

Biot-Savart law is analogous to Coulomb's law in electrostatics.

Origin of B ; $\nabla \cdot \vec{B} = 0$

For electric field $\vec{E}$, $\nabla \cdot \vec{E} = \rho/\epsilon_0$ where ρ is the electric charge density. In analogy, $\nabla \cdot \vec{B} = k_m \rho_m$, where k_m is a constant depending on the choice of units and ρ_m is the magnetic charge density. But magnetic monopoles does not exists, so $\rho_m = 0$ [Free isolated magnetic monopoles do not exist]

$$\therefore \boxed{\nabla \cdot \vec{B} = 0.}$$

But experiments shows that $\nabla \times \vec{B} = \mu_0 \vec{J}$; $\mu_0 = 4\pi \times 10^{-7}$ Henry/m is the permeability of free space. $\left(\mu_0 \epsilon_0 = \frac{1}{c^2}\right)$

$$\therefore \oint_S \vec{B} \cdot \hat{n}\, ds = 0 \qquad [\because \nabla \cdot \vec{B} = 0]$$

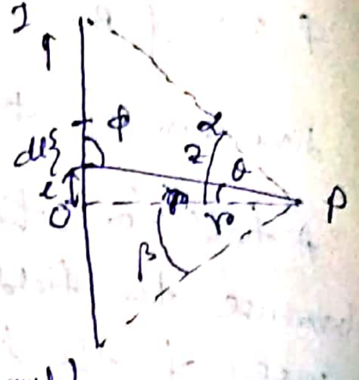
$\phi = \int \vec{B} \cdot \hat{n}\, ds$ is the magnetic flux measured in Weber.

dimension of ϕ is $[M L^2 T^{-2} I^{-1}]$

* A field for which $\nabla \cdot \vec{B} = 0$ is called a solenoidal field because the field lines close upon themselves as in the case of the field of a solenoid and there is no net flux through any closed surface.

— Application of Biot - Savart law —

① Calculation of magnetic induction($\vec{B}$) & vector potential ($\vec{A}$) for a straight wire :—



We consider a wire carrying current I.
P is a point at a distance r from it.
dl is an element of length on the wire at a distance l from O.

According to Biot - Savart law / (or Lapacce's formula) magnetic induction at P is

$$d\vec{B} = \frac{\mu_0}{4\pi} \oint \frac{I d\vec{l} \times \vec{z}}{z^3}$$

$$= \frac{\mu_0}{4\pi} \frac{I\,dl \sin\phi}{z^2} = \frac{\mu_0}{4\pi} \frac{I\,dl}{z^2} \sin\left(\frac{\pi}{2} + \theta\right)$$

$$= \frac{\mu_0}{4\pi} \frac{I\,dl \cos\theta}{z^2}$$

For the finite conductor flux density at P

$$\vec{B} = \frac{\mu_0}{4\pi} \int_{-\beta}^{\alpha} \frac{I\,dl \cos\theta}{z^2}$$

$z = r\sec\theta$
$l = r\tan\theta$
$dl = r\sec^2\theta\, d\theta$

$$= \frac{\mu_0}{4\pi} \int_{-\beta}^{\alpha} \frac{I r \sec^2\theta \cos\theta\, d\theta}{r^2 \sec^2\theta}$$

$$= \frac{\mu_0 I}{4\pi r} \int_{-\beta}^{\alpha} \cos\theta\, d\theta = \frac{\mu_0 I}{4\pi r}\left[\sin\alpha + \sin\beta\right]$$

For infinite straight conductor $\alpha = \beta = \pi/2$

$$\therefore \boxed{B = \frac{\mu_0 I}{2\pi r}}$$

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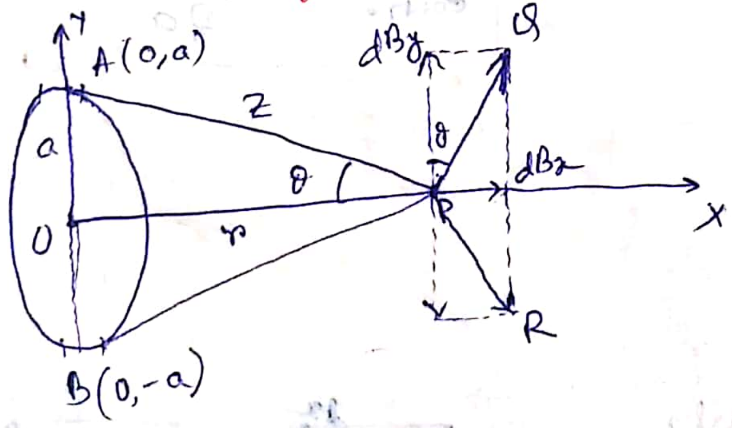
(2) Magnetic induction due to circular current loop / circular coil carrying current



We consider a circular coil of radius a carrying current I. We take an element of length dl of the coil at the point A(0, a)

From Biot - Savart law, magnetic induction at P due to the element dl is given by

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\,d\vec{l} \times \vec{r}}{r^3}$$ with magnitude $\frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}$ $\left[\because d\vec{l} \perp \vec{r}\right]$

along $\vec{PQ}$.

It has components (i) $dB_x = dB \sin\theta$

$$= \frac{\mu_0}{4\pi} \frac{I\,dl \sin\theta}{r^2}$$

(ii) $dB_y = dB\cos\theta = \frac{\mu_0}{4\pi}\frac{I\,dl\cos\theta}{r^2}$

We take an identical ~~cu~~ element of length at $B(0,-a)$ diametrically opposite to A.

Magnetic induction at P due to it is $dB = \frac{\mu_0 I\,dl}{4\pi r^2}$ along $\vec{PR}$

It also has components (i) $dB_x = dB\sin\theta = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$

(ii) $dB_y = dB\cos\theta = \frac{\mu_0}{4\pi}\frac{I\,dl\cos\theta}{r^2}$

For all such diametrically opposite elements $\oint dB_y = 0$

$$\therefore B = \oint dB_x = \frac{\mu_0}{4\pi}\int_0^{2\pi a}\frac{I\,dl\sin\theta}{r^2}$$

$\sin\theta = \frac{a}{r}$

$r = \sqrt{a^2+r^2}$

$$= \frac{\mu_0 I}{4\pi}\frac{a}{r^3}\cdot 2\pi a$$

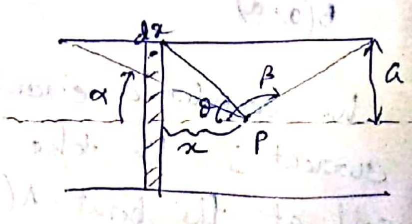
or, $$\boxed{B = \frac{\mu_0 I a^2}{2(a^2+r^2)^{3/2}}}$$

At the centre of the coil $r=0$ $\Rightarrow$ $\boxed{B_{centre} = \frac{\mu_0 I}{2a}}$

C.U/2006 (solenoid of length L)?

③ A straight solenoid

An insulated wire ~~would~~ wound closely on a cylindrical surface constitutes a solenoid. As each turn is perpendicular to the axis of the cylinder, we treat a solenoid as a number of circular coils in series.



Let the solenoid consists of N turns per unit length & carrying current I. We calculate $\vec{B}$ at a point P on its axis inside it.

We take an element of length dx of the solenoid at a distance x from P. It is a circular strip of radius a and $N dx$ turns.

$\therefore$ Magnetic induction at P due to it is

$$dB = \frac{\mu_0 I (N dx) a^2}{2(a^2+x^2)^{3/2}}$$

$$= -\frac{\mu_0 I N a^2 \, a \operatorname{cosec}^2\theta \, d\theta}{2a^3 \operatorname{cosec}^3\theta}$$

$$= -\frac{\mu_0 I N}{2} \sin\theta \, d\theta$$

$x = a\cot\theta$

$dx = -a\operatorname{cosec}^2\theta \, d\theta$

$\frac{x^2}{a^2} = \cot^2\theta$

$1+\frac{x^2}{a^2} = \operatorname{cosec}^2\theta$

For a finite solenoid, $B = -\frac{\mu_0 I N}{2}\int_\beta^\alpha \sin\theta \, d\theta$

$$= \frac{\mu_0 N I}{2}\left[\cos\alpha - \cos\beta\right] \quad Wb/m^2$$

For infinite solenoid $\alpha = 0, \beta = \pi$

$\therefore B = \mu_0 N I \quad Wb/m^2$

At the midpoint of the solenoid $\alpha = \pi - \beta$ or $\beta = \pi - \alpha$

$$\therefore B_{midpoint} = \frac{\mu_0 N I}{2}\left[\cos\alpha - \cos(\pi-\alpha)\right]$$

$$= \mu_0 N I \cos\alpha$$

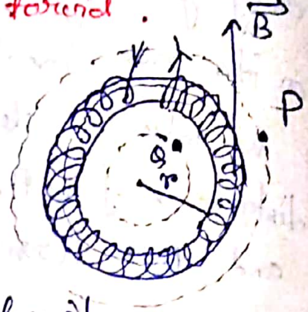
At the extreme of the solenoid, $\beta = \pi/2 \Rightarrow \cos\beta = 0$

$$\therefore B_{end} = \frac{\mu_0 N I}{2}\cos\alpha$$

Thus for a long solenoid, the value of B at one end is half that at the centre.

④

What is a toroid? Find the value of magnetic field inside, outside and within the core of a toroid.



A solenoid bent into a circular form and whose ends are put together, is called an endless solenoid or toroid.

It behaves like an infinitely long solenoid as regards magnetic field for a point within the solenoid and its magnitude is given by $B = \mu_0 NI$ where N is the no. of turns per unit length.

The lines of magnetic induction are endless circles as shown in the above fig. The magnetic field at a point inside as well as outside the core of the toroid is zero (**).

If n is the total no. of turns & r the mean radius of the toroid, then no. of turns per unit length is

$$N = \frac{n}{2\pi r}$$

$$\therefore \boxed{B = \mu_0 N I = \frac{\mu_0 n I}{2\pi r}}$$

Again $\oint \vec{B}\cdot\vec{dl} = \mu_0 i$ where i is the net current enclosed by the circle.
Now $\oint \vec{B}\cdot\vec{dl} = B(2\pi r)$ *

If r_1 & r_2 are the inner & outer radii of the core of the toroid, then,

magnetic field on the inner edge $\boxed{B_1 = \frac{\mu_0 n I}{2\pi r_1}}$

& magnetic field on the outer edge $\boxed{B_2 = \frac{\mu_0 n I}{2\pi r_2}}$

Thus the magnetic field inside the core of an ideal toroid is not uniform.

See p-132 B.S. Agarwal

* $\therefore B(2\pi r) = \mu_0 n i \Rightarrow B = \frac{\mu_0 n i}{2\pi r}$ ($n \Rightarrow$ total no. of turns)

(**) Field inside a point (such as Q) is zero because there is no current enclosed by the circle through Q. Similarly the field at a point such as P is also zero, because each turn of the winding passes twice through the area enclosed by the circle through P, carrying equal currents in opposite directions, so that the net current enclosed by this circle is zero.

~~Ampere's theorem~~ / Ampere's circuital law

The lines of magnetic field are continuous and do not arise from any source in the way as lines of electric force originate on charges. These lines of forces thus form loops without any beginning or end. This property is used to calculate the magnetic field for a symmetric current distribution.

Ampere's circuital law states that: The line integral of the magnetic induction B around any closed path is equal to μ_0 times the net current across the area bounded by this path.

i.e, $\boxed{\oint \vec{B}.\vec{dl} = \mu_0 I}$

It plays the same role in magnetostatics as that Gauss's law $\left(\int \vec{E}.\vec{ds} = q/\epsilon_0\right)$ played in electrostatics.

Proof

We have the field round a long wire carrying a current I is $B = \frac{\mu_0 I}{2\pi r}$. The magnetic lines are concentric around the wire in the plane of the paper if the wire is $\perp$ to this plane. The line integral around the path of radius r, starting at any point and returning to the same point is

$$\oint \vec{B}.\vec{dl} = \int B.dl = \int \frac{\mu_0 I}{2\pi r} dl = \frac{\mu_0 I}{2\pi r}\int_0^{2\pi r} dl$$

$$= \frac{\mu_0 I}{2\pi r}.2\pi r = \mu_0 I$$

$$\therefore \boxed{\oint \vec{B}.\vec{dl} = \mu_0 I}$$ → Integral form of Ampere's law

So, the line integral does not depend on the shape of the path or on the position of the wire.

If any number of long straight conductors pass through the surface bounded by the path, the line integral equals μ_0 times the algebraic sum of the currents.

According to Ampere's circuital law.

$$\oint \vec{B}.\vec{dl} = \mu_0 I$$

or, $$\oint \vec{H}.\vec{dl} = I \quad \left[\because \vec{B} = \mu_0 \vec{H}\right]$$

Again $$\int (\nabla \times \vec{B}) \vec{ds} = \mu_0 I = \mu_0 \int \vec{J}.\vec{ds} \quad \left[\because I = \int \vec{J}.\vec{ds}\right]$$

or, $$\boxed{\nabla \times \vec{B} = \mu_0 \vec{J}}$$ differential form of circuital law

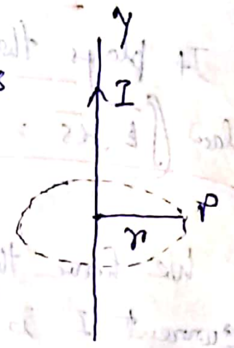
or $$\nabla \times \vec{H} = \vec{J}$$

Ampere's law is always true as Gauss's law. But it can be applied only for uniform & symmetrical current distribution. (Tewari p-322)

v/2006

Application of Ampere's circuital law :

Magnetic field due to current carrying straight conductor of infinite length



We consider an infinitely long wire along Y-axis carrying current I.

The lines of magnetic induction will consist of concentric circles with the wire as the centre. Let $\vec{B}$ be the magnetic induction field at a point P at a dist. r from the wire.

According to Ampere's law, the line integral of the magnetic field for a closed path $\oint \vec{B}.\vec{dl} = \mu_0 I$

From symmetry the magnitude of B is the same at all points lying at a distance r from the wire, i.e, on the circle of radius r.

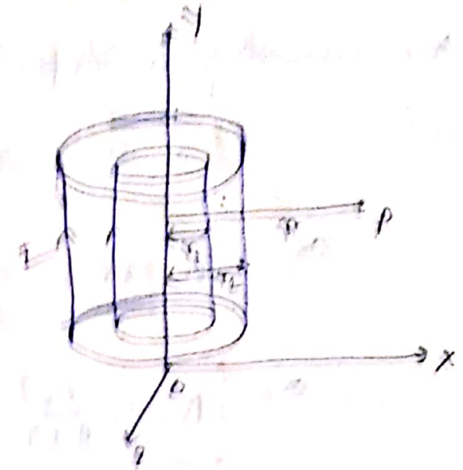
$$\therefore \oint \vec{B}.\vec{dl} = B\,2\pi r$$

or, $$2\pi B r = \mu_0 I$$

or $$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

Calculate the magnetic induction due to an infinite hollow tube carrying a steady current at points inside & outside the tube.

Let r_1 & r_2 be the internal & external radii of the hollow tube. A uniform current I is flowing in it along the periphery of the tube in the $+Y$ direction. From symmetry, we have, the lines of magnetic induction are concentric circles around the axis of the tube, in the counter clockwise direction.



For a point outside the tube — $(r > r_2)$

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I \Rightarrow B.2\pi r = \mu_0 I \Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi r}}$$

Hence the mag. field at a point outside the cylindrical tube is the same as if all the current were flowing along the axis.

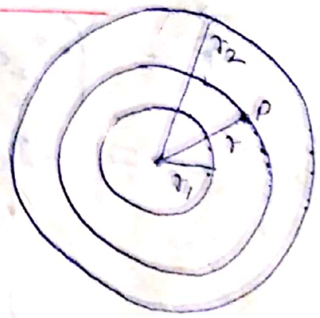
For a point inside the tube $(r < r_1)$ —

Here P lies inside the inner hollow tube & no current is enclosed by a circular path of radius r around the axis of the tube.

$$\therefore \; I = 0 \Rightarrow B\cdot 2\pi r = 0 \Rightarrow \boxed{B = 0}$$

For a point bet^n the inner & outer tube $(r_1 < r < r_2)$ -

In this case only the part of the current flowing in the tube of inner radius r_1 & outer radius r will be effective in producing the magnetic field at this point. The current flowing in the tube of inner radius r & outer radius r_2 will not produce any magnetic field at a point inside r.



The area of the annular face of the whole tube $= \pi(r_2^2 - r_1^2)$

The area of the annular face of the tube of internal & external radii r_1 & r $= \pi(r^2 - r_1^2)$

$\therefore$ Part of the current flowing through the annular face of this tube

$$= \frac{\pi(r^2 - r_1^2)}{\pi(r_2^2 - r_1^2)} I = \left(\frac{r^2 - r_1^2}{r_2^2 - r_1^2}\right) I$$

Now according to Ampere's law

$$\oint \vec{B}\cdot\vec{dl} = \mu_0\left(\frac{r^2-r_1^2}{r_2^2-r_1^2}\right) I$$

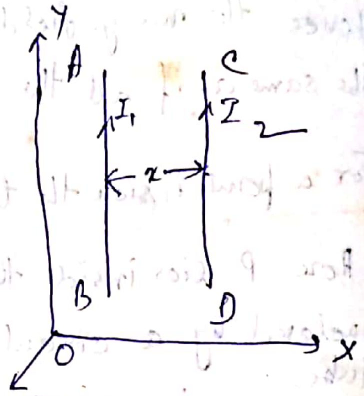
or,

$$B\,2\pi r = \mu_0\left(\frac{r^2-r_1^2}{r_2^2-r_1^2}\right) I$$

or

$$\boxed{B = \frac{\mu_0 I}{2\pi r}\left(\frac{r^2-r_1^2}{r_2^2-r_1^2}\right)}$$

Force between long parallel current carrying conductors



AB & CD are two long straight conductors carrying currents I_1 & I_2.

According to Biot-Savart's law, the intensity of the magnetic field $\vec{B}$ at a point on the wire CD due to the current I_1 in AB is given by

$$\vec{B} = -\frac{\mu_0 I_1}{2\pi x}\hat{k} \qquad \left[\because B = \int \frac{I_1\,\vec{dl}\times\vec{r}}{r^3} \quad \hat{j}\times\hat{i} = -\hat{k}\right]$$

Force experienced by a length $\vec{dl}$ of the wire CD due to field $\vec{B}$

$$\vec{F} = \int I_2\,\vec{dl}\times\vec{B}$$

$$= -I_2\frac{\mu_0 I_1}{2\pi x}(\hat{j}\times\hat{k}) = (-\hat{i})\frac{\mu_0 I_1 I_2}{2\pi x}\int_0^l dl$$

$$\boxed{\vec{F} = (-\hat{i})\frac{\mu_0 I_1 I_2 l}{2\pi x}}$$

Hence force per unit length $= \frac{\mu_0 I_1 I_2}{2\pi x}(-\hat{i})$

$(-\hat{i})$ indicates that this is the force of attraction

The force is mutual in accordance with Newton's third law of motion.

For parallel current the wires will attract & for antiparallel current they will repel each other.